

## Math 2200-1. Quiz 3. Solutions. Fall 2008.

December 9, 2008

**Problem 1.** What is the probability that a seven-card poker hand contains:

- (a) four cards of one kind and three cards of a second kind?
- (b) pairs of each of three different kinds and a single card of a fourth kind?
- (c) a flush (all cards of the same suit)?

*Proof.* (a)  $\frac{P(13,2)C(4,4)C(4,3)}{C(52,7)}$ . Note that we need  $P(13,2)$ , not  $C(13,2)$ , since it matters which kind gives four cards and which gives three.

(b)  $\frac{C(13,3)C(4,2)^3C(10,1)C(4,1)}{C(52,7)}$ . The three kinds giving the pairs are indistinguishable as far as the order goes. The fourth kind, on the other hand has to be chosen from the remaining 10 kinds.

(c)  $\frac{C(4,1)C(13,7)}{C(52,7)}$ . First we choose a suit, and then 7 out of 13 cards in that suit.  $\square$

**Problem 2.** Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled? (You should give a rigorous proof, by computing the probabilities involved.)

*Proof.* For two dice, the number of favorable cases is 5: (6, 2), (5, 3), (4, 4), (3, 5), (2, 6), so the probability is  $\frac{5}{36} = \frac{30}{216}$ .

For three dice, the number of favorable cases is 21: (5, 2, 1), (4, 3, 1) counted with all permutations (so  $6 \cdot 2 = 12$ ), and (1, 1, 6), (2, 2, 4), (3, 3, 2), counted with the distinct permutations (so  $3 \cdot 3 = 9$ ). The probability is  $\frac{21}{216}$ .

So the more likely case is for two dice.  $\square$

**Problem 3.** There are three cards in a box. Both sides of one card are black, both sides of one card are red, and the third card has one black side and one red side. We pick a card at random and observe only one side.

- (a) If the side is black, what is the probability that the other side is also black?
- (b) What is the probability that the opposite side is the same color as the one we observed?

*Proof.* (a) Let  $F$  be the event that the side we observe is black, and  $E$  the event that the other side is also black. We want the conditional probability  $P(E|F) = \frac{P(E \cap F)}{P(F)}$ . The event  $E \cap F$  means both sides are black, and there is only one card like that out of three, so  $p(E \cap F) = \frac{1}{3}$ . For the

event  $F$  we count the sides: there are six sides, three black, three red, so  $p(F) = \frac{1}{2}$ . Therefore the answer is  $p(E|F) = \frac{2}{3}$ .

(b) Let  $E'$  be the event that both sides have the same color. There are two cards out of three which are favorable, so  $p(E') = \frac{2}{3}$ .

□