

Math 2200-1. Quiz 2. Fall 2008.

Name: _____

November 20, 2008

Problem 1: _____ /20

Problem 2: _____ /20

Problem 3: _____ /20

Problem 4: _____ /20

Problem 5: _____ /20

Total: _____ /100

Instructions: The exam is closed book, closed notes and calculators are not allowed. You are only allowed one letter-size sheet of paper with anything on it.

You will have 60 minutes for this test. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

Problem 1. Let f_n , $n \geq 1$ denote the Fibonacci numbers. Recall that they are defined recursively as follows: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, for $n \geq 2$.

- (1) Find f_7 .
- (2) Prove by induction that

$$f_1 + f_2 + \cdots + f_n = f_{n+2} - 1,$$

for all $n \geq 1$.

Problem 2. Prove by induction that

$$2n + 3 \leq 2^n,$$

for all $n \geq 4$.

Problem 3. Prove by induction that 3 divides $n^3 + 2n$ for all positive integers n .

Problem 4. How many permutations of the letters $ABCDEFGH$:

- (1) contain the string CDF ?
- (2) contain the string FH ?
- (3) contain the string EF ?
- (4) do not contain any of the strings CDF , FH , and EF ?

Problem 5.

- (1) According to wikipedia, the population of New York State is 19 million. What must be the minimum number of people in the state of New York with the same three initials who were born on the same day of the year (but not necessarily in the same year)? Assume that everyone has three initials. Explain your answer.
- (2) Assume all the license plates in NY consist of three letters followed by three digits. How many different license plates are possible if the requirement is that a license plate contains no letter twice *or* no digit twice?

Extra credit problem. *Attempt to solve it only if you finished the first five problems!*

Show that in any set of $n + 1$ positive integers, not exceeding $2n$ there must be two that are relatively prime.