

Math 2200-1. Quiz 1. Fall 2008.

Name: _____

September 25, 2008

Problem 1: _____ /15

Problem 2: _____ /15

Problem 3: _____ /17

Problem 4: _____ /18

Problem 5: _____ /18

Problem 6: _____ /17

Total: _____ /100

Instructions: The exam is closed book, closed notes and calculators are not allowed. You are only allowed four letter-size sheets of paper with anything on it.

You will have 75 minutes for this test. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

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Problem 1. (15 points) Show carefully that the compound proposition

$$((p \vee q) \wedge \bar{p}) \rightarrow q$$

is a tautology.

Problem 2. (15 points) Is the following argument valid? (Carefully express it in propositional logic, and show the rules of inference used at each step.)

“You can score well in the GRE only if you have good analytical skills. Every student who takes Discrete Math has good analytical skills or good memory. Maggie doesn’t have good memory. Therefore, if Maggie takes Discrete Math, then Maggie will score well in the GRE.”

Problem 3. (17 points)

(a) Express the following sentence using quantifiers (and logical propositions): *“There is a student in this class who has taken some course in every department in the school of science.”*

You should use the variables: s student, c course, d department, and these should be the only variables. Give the domain of each variable.

(b) Find the logical negation of the quantifier expression that you obtained in (a).

(c) Translate the negation you obtained in (b) into an English sentence.

Problem 4. (18 points)

(a) List the elements of the set $S = \{x \mid x \in \mathbb{Z} \text{ and } (x + 1)(2 - x) \geq 0\}$.

(b) For every integer $i \geq 1$, set $A_i = \{-i, \dots, -1, 0, 1, 2, 3, \dots\}$. Find

$$A_{100} - A_{96}, \quad \bigcup_{i=1}^{\infty} A_i, \quad \bigcap_{i=1}^{\infty} A_i.$$

Problem 5. (18 points)

(a) Is the function $f : \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$, given by $f(x) = 3x^2 + 2$ one-to-one? What is its image(range)?

(b) Consider $g : \mathbb{R} \rightarrow \mathbb{Z}$, $g(x) = \lfloor x + \frac{1}{2} \rfloor$. What is the preimage $g^{-1}(\{-1, 1\})$?

Problem 6. (17 points) Prove the following statements:

(a) For every $x > 0$, if x is irrational, then \sqrt{x} is irrational.

(b) Between any two distinct rational numbers, there exist infinitely many rational numbers.