Math 1060. Trigonometry. Lecture 12.10.09

Review. The following problems are taken from the textbook pages 487-488. If you need to verify your answers, check the back of the book.

(1) Consider the angle $\theta = -120^\circ$.
   (a) Sketch the angle in standard position.
   (b) Determine the coterminal angle in the interval $[0^\circ, 360^\circ)$.
   (c) Convert the angle to radian measure.
   (d) Find the reference angle $\theta'$.
   (e) Write the exact values of the six trigonometric functions of $\theta$.

(2) Convert the angle $\theta = \frac{3\pi}{5}$ from radians to degrees.

(3) Find $\cos \theta$ if $\tan \theta = -\frac{4}{3}$ and $\sin \theta < 0$.

(4) Sketch the graph of the function
   (a) $f(x) = 3 - 2 \sin \pi x$
   (b) $g(x) = \frac{1}{2} \tan(x - \frac{\pi}{2})$
   (c) $h(x) = \sec(x + \pi)$

(5) Find the exact value of the expressions:
   (a) $\arccos(-\frac{\sqrt{3}}{2})$
   (b) $\arcsin(-\frac{\sqrt{2}}{2})$
   (c) $\tan(\arctan 6.7)$
   (d) $\tan(\arcsin \frac{4}{5})$
   (e) $\sin(\arccos \frac{2}{3})$

(6) Use the fundamental identities to simplify $\cos(\frac{\pi}{2} - x) \csc x$

(7) Subtract and simplify $\frac{\sin \theta - 1}{\cos \theta} - \frac{\cos \theta}{\sin \theta - 1}$

(8) Verify the identities
   (a) $\cot^2 \alpha (\sec^2 \alpha - 1) = 1$
   (b) $\sin^2 x \cos^2 x = \frac{1}{8}(1 - \cos 4x)$
   (c) $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

(9) Find all solutions in the interval $[0, 2\pi)$
   (a) $2 \cos^2 \beta - \cos \beta = 0$
   (b) $3 \tan \theta - \cot \theta = 0$
   (c) $\sin^2 x + 2 \sin x + 1 = 0$

(10) Given that $\sin u = \frac{12}{13}$, $\cos v = \frac{3}{5}$, and the angles $u$ and $v$ are both in Quadrant I, find $\tan(u - v)$.

(11) If $\tan \theta = \frac{1}{2}$, find the exact value of $\tan(2\theta)$.

(12) Solve the triangle:
(a) $A = 30^\circ$, $a = 9$, $b = 8$
(b) $A = 30^\circ$, $b = 8$, $c = 10$
(c) $A = 30^\circ$, $C = 90^\circ$, $b = 10$
(d) $a = 4$, $b = 8$, $c = 9$

(13) Two sides of a triangle have lengths 7 inches and 12 inches. Their included angle measures $60^\circ$. Find the area of the triangle.

(14) Find a unit vector in the direction of $v = i + 2j$.

(15) Find $u \cdot v$ for $u = 3i + 4j$ and $v = i - 2j$. Find $u + v$ and draw it in the standard position.

(16) Find the projection of $u = \langle 8, -2 \rangle$ onto $v = \langle 1, 5 \rangle$. Then write $u$ as a sum of two orthogonal vectors.

(17) Write the complex number $-2 + 2i$ in trigonometric form and compute its 8-th power.

(18) A ceiling fan with 21-inch blades makes 63 revolutions per minute. Find the angular speed of the fan in radians per minute. Find the linear speed of the tips of the blades in inches per minute.

(19) To determine the angle of elevation of a star in the sky, you get the star in your line of vision with the backboard of a basketball hoop that is 5 feet higher than your eyes. Your horizontal distance from the backboard is 12 feet. What is the angle of elevation of the star?

(20) Convert the point:
(a) $(2, 1)$ from rectangular to polar coordinates
(b) $(-3, \frac{\pi}{6})$ from polar to rectangular coordinates.

(21) Convert the rectangular equation to polar form:
(a) $x^2 + y^2 = 49$
(b) $xy = 5$

(22) Convert the polar equation to the rectangular form:
(a) $r = 5$
(b) $r = 3 \cos \theta$
(c) $r^2 = \sin \theta$