Review for Test 2

- Definition of a vector space.
  1. What is a subspace? How to check when a subset of a vector space is a subspace?
- Dimension, Bases
  1. Intuitive idea of dimension.
  2. What is a basis?
  3. What is the span of a collection of vectors?
  4. What is a linear combination?
  5. What is linear independence? How to check for linear independence?
  6. Examples of finite-dimensional vector spaces. Of infinite-dimensional vector spaces.
  7. The number of elements of ANY spanning set is always greater or equal to the dimension of a vector space. Why?
- Linear maps
  1. Examples of linear maps. Examples of non-linear maps.
  2. How to check linearity of a given map?
  3. What is an isomorphism (of vector spaces)?
  4. What is injectivity? How to check injectivity for a given linear map?
  5. Does injectivity and surjectivity imply one another in general? Examples of the failure of the above implication in either direction. What condition have you learned that guarantees their equivalence?
  6. Kernels and images of linear maps in $\mathrm{L}(V, W)$ are subspaces (of what vector space?)
  7. The Rank-Nullity Theorem. Remember this is just a familiar fact about systems of linear equations expressed in vector-space language. Examples of failure of the Theorem in infinite dimensions.
  8. The collection of linear maps from one vector space to another forms a vector space. The collection of linear maps of a vector space to itself forms an algebra where multiplication is defined to be composition.
- Matrix representatives of vectors and linear maps:
  1. In how many ways can you express a given vector as a linear combination of the basis vectors of a given basis?
  2. How does this give rise to matrix representatives of vectors and linear maps? In other words, how to form matrix representatives?
  3. Transformation laws for matrix representatives (for both vectors and linear maps).
  4. Change-of-basis matrices are invertible matrices.
  5. The action of a linear map is “represented” by left multiplication by its matrix representative (w.r.t. appropriate bases).
  6. Taking matrix representative (w.r.t. fixed chosen bases) is an isomorphism:
     a) Matrix representatives are unique (injectivity).
     b) Every matrix represents a vector or linear map (surjectivity).
     c) The matrix representative of a sum (of vectors or linear maps) is the sum of the representatives.
     d) The matrix representative of a scalar multiple (of a vector or a linear map) is the scalar multiple of the representative.
  7. The matrix representative of the composition of two linear maps is the matrix product of the matrix representatives of the individual linear maps.