Comparing Matrix and Complex-Number Arithmetics

	$\alpha, \beta, \gamma \in \mathbb{C}$	$A \in \mathbb{C}^{m\times n}, B \in \mathbb{C}^{p\times q}, C \in \mathbb{C}^{r\times s}$
Addition	$\alpha + \beta$ always defined Commutative: $\alpha + \beta = \beta + \alpha$ Associative: $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ Zero: $\alpha + 0 = \alpha$ Additive inverse: $\alpha + (-\alpha) = 0$	$A + B$ defined only when $m = p$ and $n = q$ Commutative: $A + B = B + A$ Associative: $(A + B) + C = A + (B + C)$ Zero: $A + 0_{m\times n} = A$ Additive inverse: $A + (-1 \cdot A) = 0_{m\times n}$
Scalar Multiplication	Same as multiplication; see below.	$\alpha \cdot A$ always defined "Commutative": $\alpha \cdot A = A \cdot \alpha$ "Associative": $\alpha \cdot (\beta \cdot A) = (\alpha\beta) \cdot A$
Multiplication	$\alpha\beta$ always defined Commutative: $\alpha\beta = \beta\alpha$ Associative: $(\alpha\beta)\gamma = \alpha(\beta\gamma)$ Identity: $\alpha \cdot 1 = \alpha = 1 \cdot \alpha$ Multiplicative inverse: $\alpha \cdot (\frac{1}{\alpha}) = 1 = (\frac{1}{\alpha}) \cdot \alpha$, for $\alpha \neq 0$	$A \cdot B$ defined only when $n = p$ NOT Commutative: $A \cdot B \neq B \cdot A$, in general Associative: $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ Identity exists for SQUARE matrices only: $A \cdot I_n = A = I_n \cdot A$, for $A \in \mathbb{C}^{n\times n}$ Multiplicative inverse: A^{-1} exists only for $A \in \mathbb{C}^{n\times n}$ with $\det(A) \neq 0$, and for those, we have $A \cdot A^{-1} = I_n = A^{-1} \cdot A$ (much much more on determinants)
Distribution	$\alpha\,(\beta + \gamma) = \alpha\beta + \alpha\gamma = (\beta + \gamma)\,\alpha$	$(\alpha + \beta) \cdot A = \alpha \cdot A + \beta \cdot A = A \cdot (\alpha + \beta)$ $\alpha \cdot (A + B) = \alpha \cdot A + \alpha \cdot B = (A + B) \cdot \alpha$ $A \cdot (B + C) = A \cdot B + A \cdot C$ $(B + C) \cdot A = B \cdot A + C \cdot A$