The Graph of an $\mathbb{R} \longrightarrow \mathbb{R}$ Function

Let X be a subset of $\mathbb{R}$ and $f : X \longrightarrow \mathbb{R}$ be a given function. We seek to visually represent f.

Definition The *graph* of $f : X \longrightarrow \mathbb{R}$ is defined to be

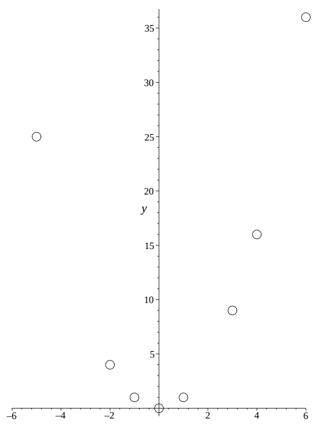
$$\text{Graph}(f) := \{(x, y) \in \mathbb{R}^2 \mid x \in X \text{ and } y = f(x)\},$$

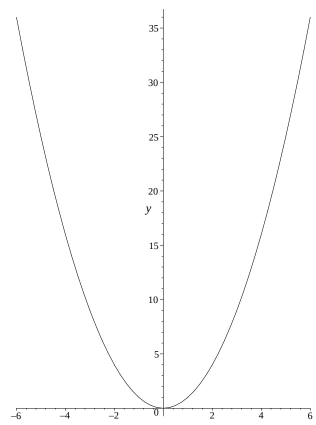
i.e. Graph(f) is the set of all ordered pairs of real numbers of the form $(x, f(x))$, where $x \in X = \text{Domain}(f)$.

By definition, Graph(f) is subset of $\mathbb{R}^2$. Recall now that the Cartesian plane is a graphical representation of $\mathbb{R}^2$. Thus, "to plot the graph of the function f" means to plot the set of points on the Cartesian plane that correspond to the subset Graph(f) of $\mathbb{R}^2$.

Example

Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ with $f(x) = x^2$. Then some elements of Graph(f) are: $(-5, 25)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(3, 9)$, $(4, 16)$, $(6, 36)$. We can plot all these points on the Cartesian plane as in the diagram on the left below:





Moreover, if we plot *all* the points instead, we get a curve as in the diagram on the right.

The Vertical Line Test

Given a curve in the xy-plane, if it intersects every vertical line at at most one point, then it is the graph of a function with independent variable x and dependent variable y. Why?
Hint: It has to do with the fact that functions are by definition "single-valued", namely, every element in the domain corresponds to exactly one element in the codomain. (See also p.79 of textbook.)

Exercises

1. Explain why each of the following curves does or does not represent a function. For each one, consider both the possiblilities of y as a function of x *and* x as a function of y.