1. When the wind blows over a chimney, vortices are shed into the wake. The frequency of vortex shedding $f$ depends on the chimney diameter $D$, its length $L$, the wind velocity $V$ and the kinematic viscosity of air $\nu$. Express the nondimensional shedding frequency in terms of its dependence on the other nondimensional groups.

2. A cone and plate viscometer consists of a cone with a very small angle $\alpha$ which rotates above a flat surface. The torque required to spin the cone at a constant speed is a direct measure of the viscous resistance. The torque $T$ is a function of the radius $R$, the cone angle $\alpha$, the fluid viscosity $\mu$, and the angular velocity $\omega$.
   (a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups.
   (b) If $\alpha$ and $R$ are kept constant, how will the torque change if both the viscosity and the angular velocity are doubled?

3. Two incompressible viscous fluids of the same density $\rho$ flow, one on top of the other, down an inclined plane making an angle $\alpha$ with the horizontal. Their viscosities are $\mu_1$ and $\mu_2$, the lower fluid is of depth $h_1$ and the upper fluid is of depth $h_2$. Show that
   \[ u_1(y) = \left[(h_1 + h_2)y - \frac{1}{2}y^2\right] \frac{g \sin \alpha}{\nu_1}. \]

4. A viscous flow is generated in $r \geq a$ by a circular cylinder $r = a$ which rotates with constant angular velocity $\Omega$. There is also a radial inflow which results from a uniform suction on the (porous) cylinder, so that $u_r = -U$ on $r = a$. Show that
   \[ u_r = -\frac{U a}{r} \quad \text{for} \quad r \geq a, \]
   and that
   \[ r^2 \frac{d^2 u_\theta}{dr^2} + (\text{Re} + 1) r \frac{du_\theta}{dr} + (\text{Re} - 1) u_\theta = 0, \]
   where $\text{Re} = U a / \nu$. Show that if $\text{Re} < 2$ there is just one solution of this equation which satisfies the no-slip condition on $r = a$ and has finite circulation $\Gamma = 2\pi r u_\theta$ at infinity, but that if $\text{Re} > 2$ there are many such solutions.
   The circulation around a cylinder of radius $a$ is $\Gamma = \oint u \cdot dr = \oint u_\theta a d\theta = 2\pi u_\theta a$.

5. Consider two parallel plates located at $y = \pm L$. Assume that the pressure gradient in the $x$-direction oscillates in time, i.e. $\frac{\partial P}{\partial x} = P_x \cos(nt)$, where $P_x$ is constant representing the magnitude of the pressure-gradient oscillations. Assuming no-slip and no-penetration boundary conditions and one dimensional flow, show that the solution to this unsteady problem is
   \[ u(y, t) = \text{Re} \left( i \frac{P_x}{\rho n} \left[1 - \frac{\cosh \left(\frac{(1 + i)n}{(2\nu)L}\right)}{\cosh \left(\frac{(1 + i)n}{(2\nu)L}\right)}\right] e^{int}\right) \]

6. Show that the dispersion relation for waves on the interface between two fluids, the upper fluid being of density $\rho_2$ and the lower being of density $\rho_1$ with $\rho_1 > \rho_2$ is
   \[ c^2 = \frac{\omega^2}{k^2} = \frac{g}{k} \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right). \]