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Numerical Methods for Sloshing Dynamics with Surface Tension

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Mathematics Department Award Ceremony

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Special Thanks: Chee Han Tan and Hari Sundar (CS)

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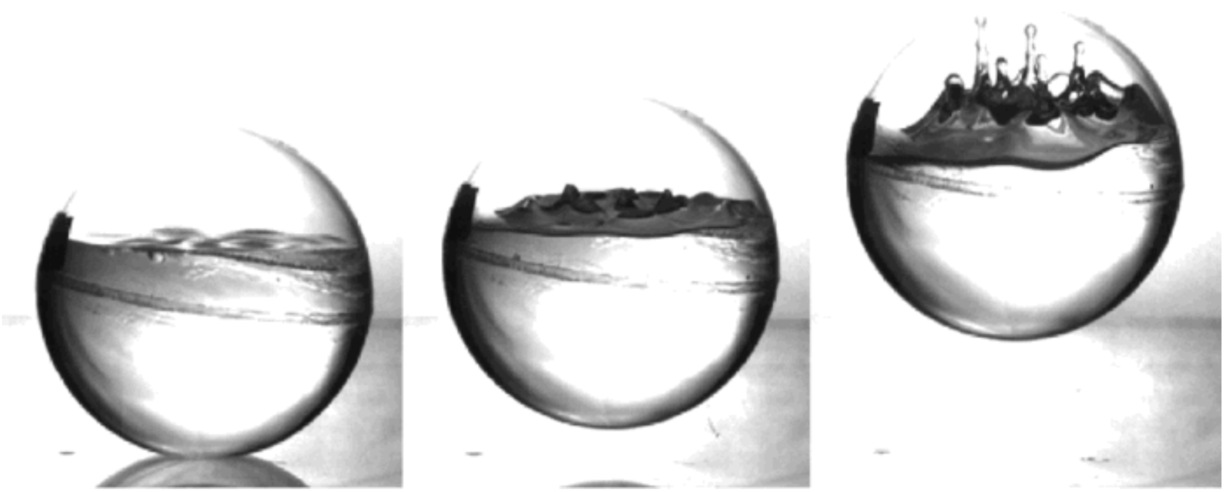
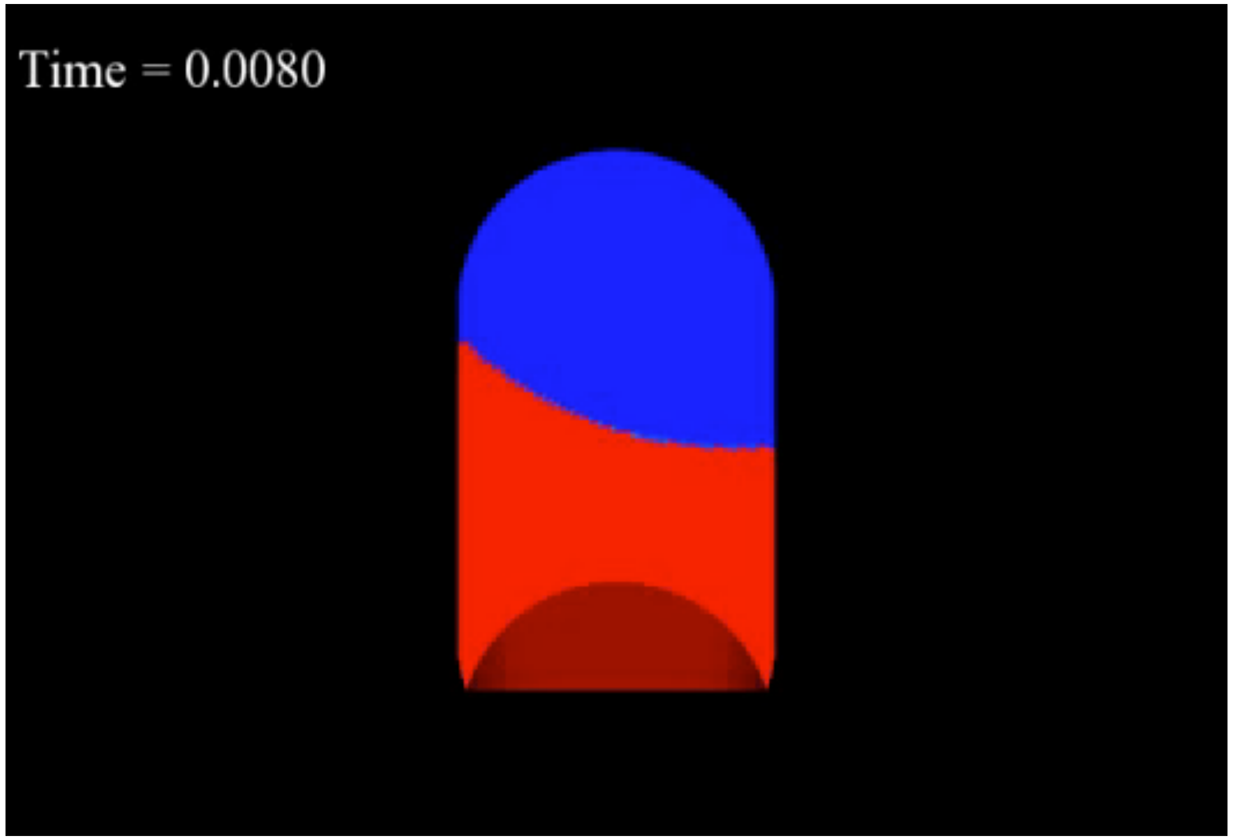
Motivation



Utah State University Splash Lab



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- Sloshing dynamics is the study of the motion of a liquid in a container.
- Focus on the regime where the effect of surface tension dominates the effect of gravity.
- Surface tension results from cohesive force between liquid molecules on a surface.

Linearized Sloshing Dynamics Model

- Look for time periodic solutions. Assume small oscillations.
- Solve for the velocity potential $\phi(x, y, z)$, the surface height $\eta(x, y)$ and the sloshing frequency ω.

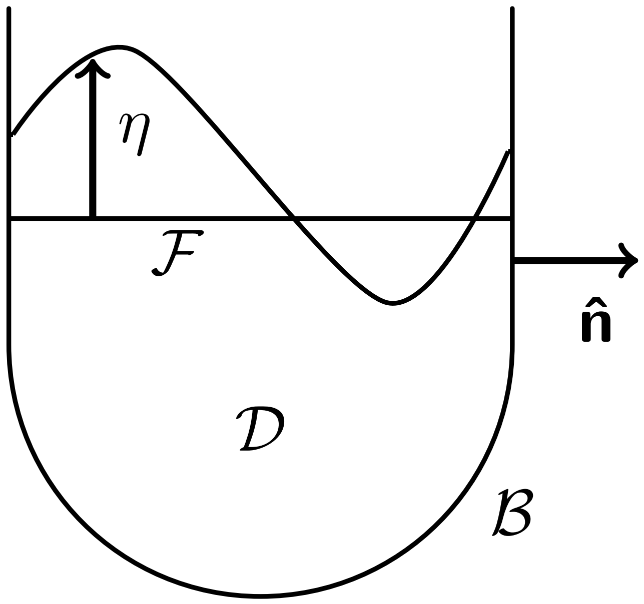
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad \text{in } \mathcal{D}$$

$$\nabla \phi \cdot \hat{\mathbf{n}} = 0 \qquad \text{on } \mathcal{B}$$

$$\frac{\partial \phi}{\partial z} = \omega\, \eta \qquad \text{on } \mathcal{F}$$

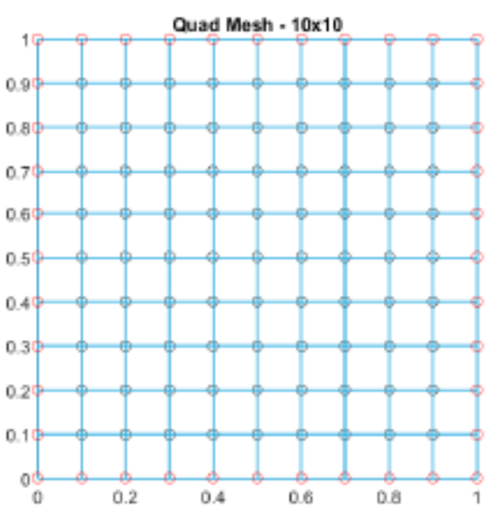
$$\eta - \frac{1}{\boxed{\mathrm{Bo}}} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) = \omega\, \phi \qquad \text{on } \mathcal{F}$$

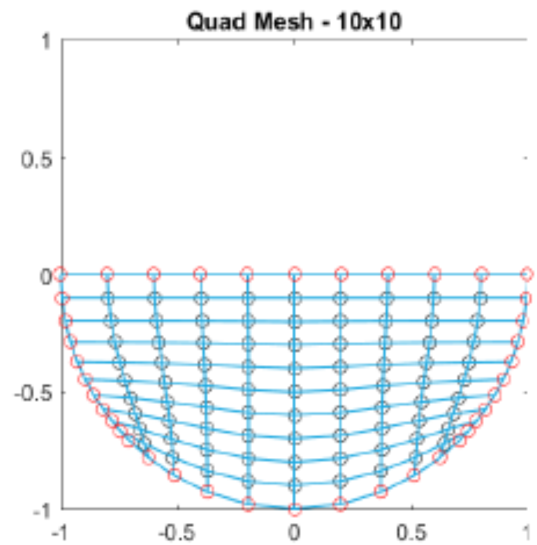
$$\nabla \eta \cdot \hat{\mathbf{n}} = 0 \qquad \text{on } \mathcal{F} \cap \mathcal{B}$$

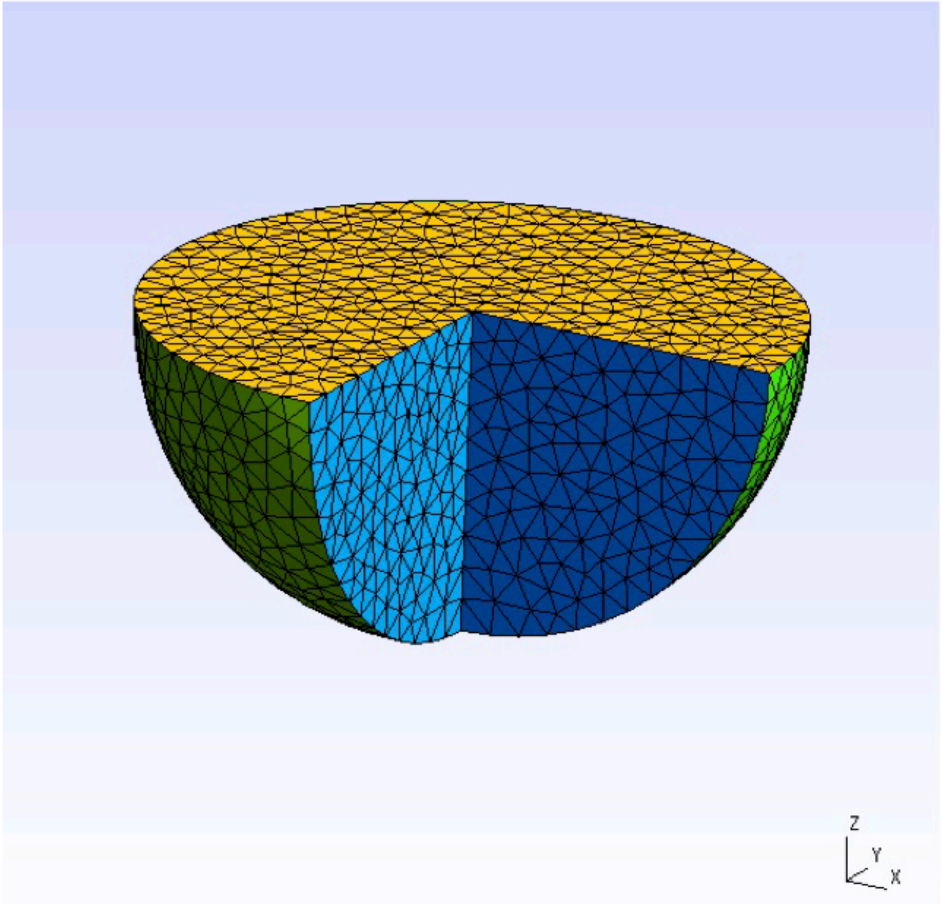


Bond number: $\mathrm{Bo} = \dfrac{\rho g L^2}{T}$ ratio of gravitational force to surface tension force

Numerical Method

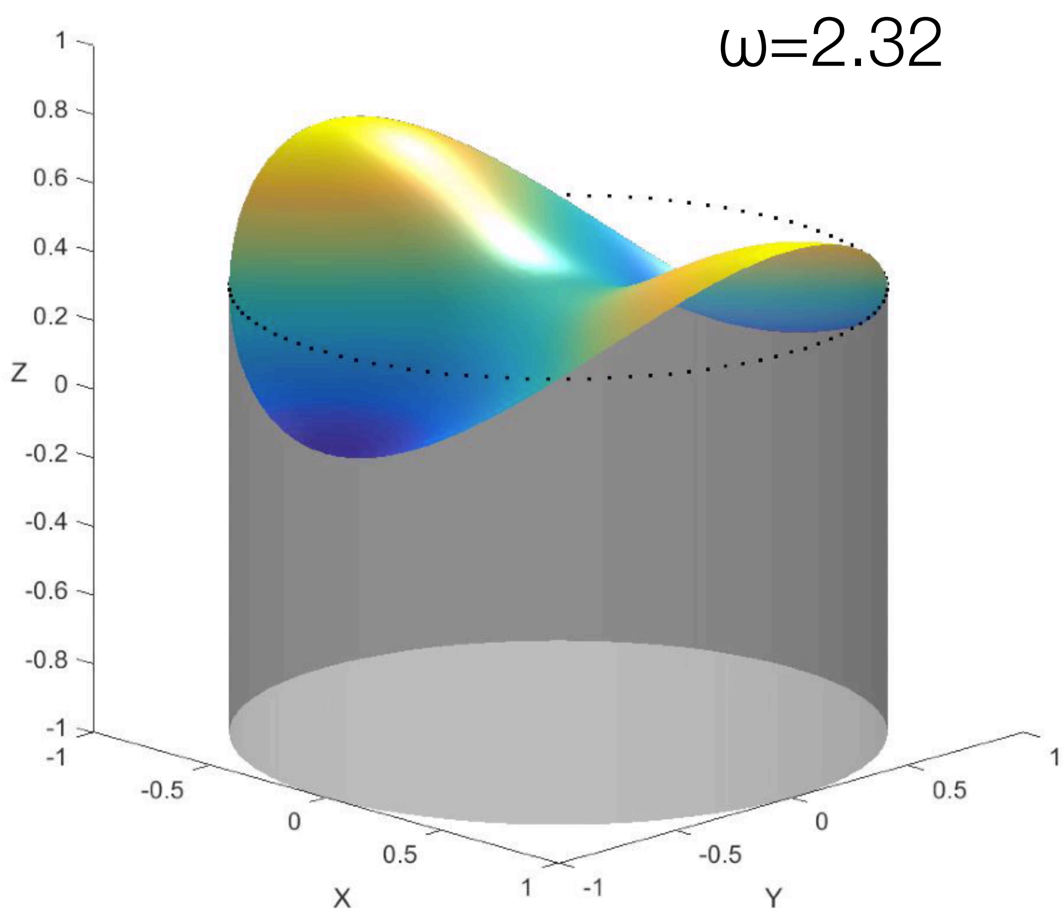


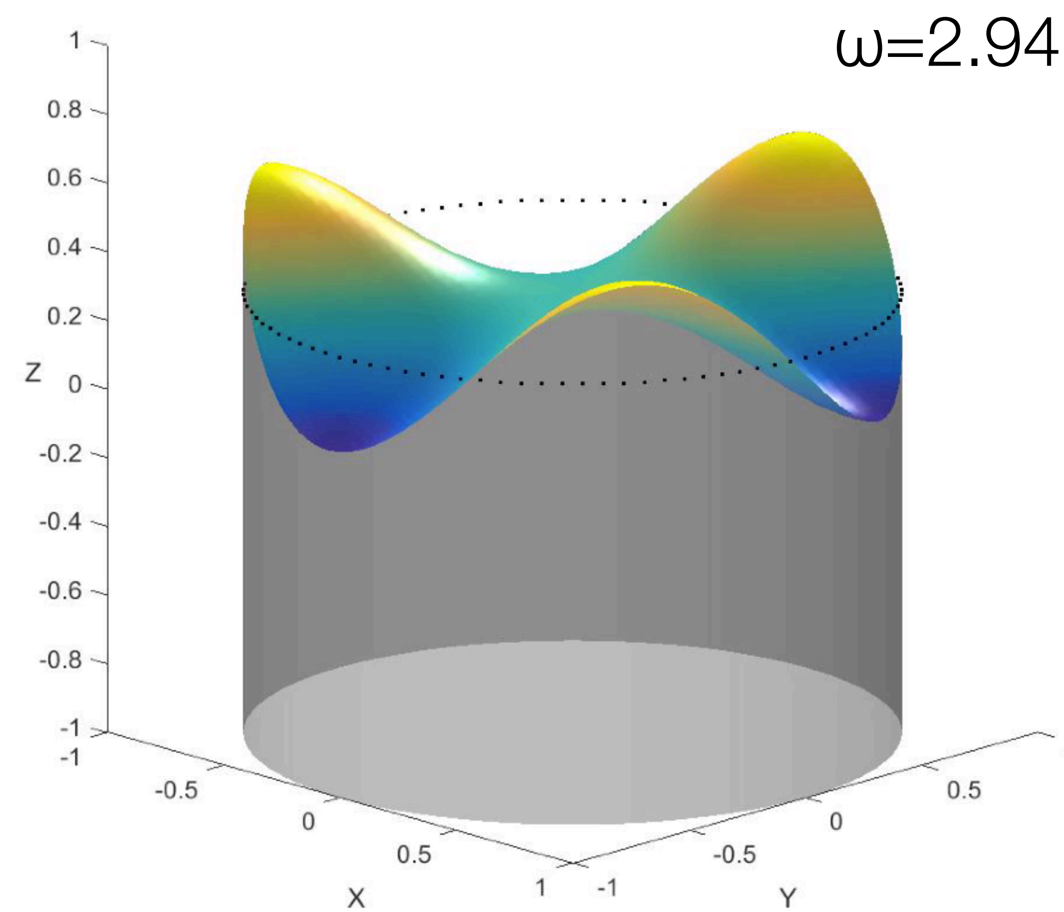




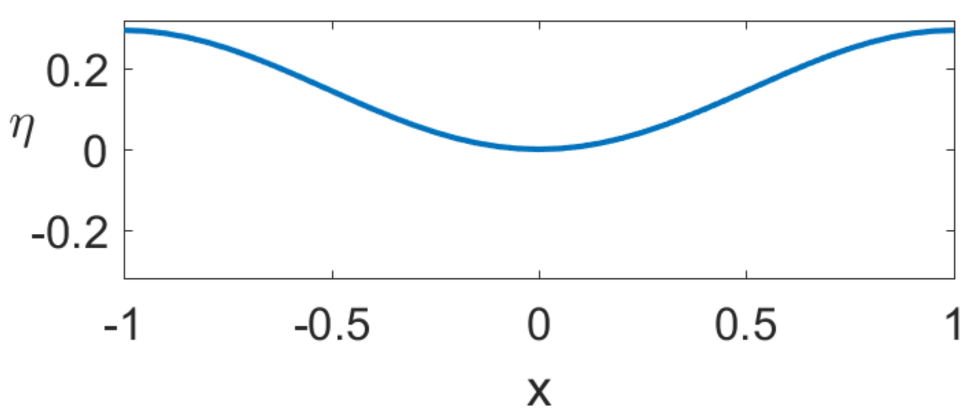
- Initialize a mesh grid on a cube and transform it to the desired container shape.
- Approximate solutions using a finite-element method (FEM), essentially solving a linear algebra problem.
- Use **Hari Sundar's hexahedral solver, Matlab version.** Limited mesh size.

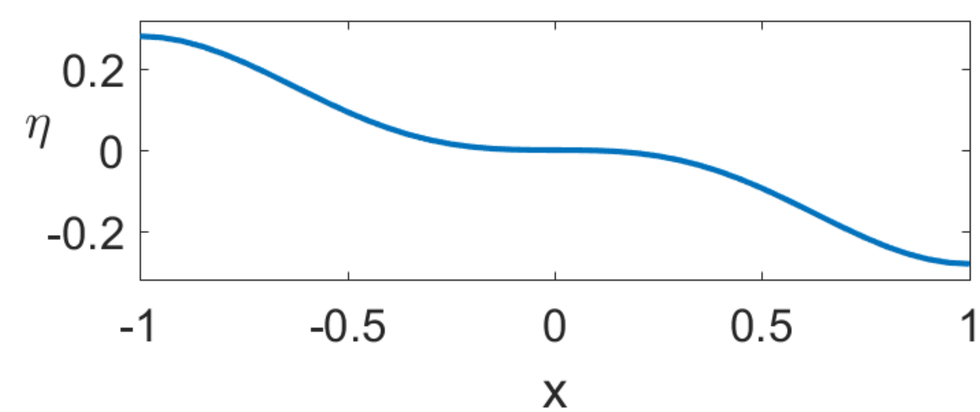
Results: Cylinder, Bo=10



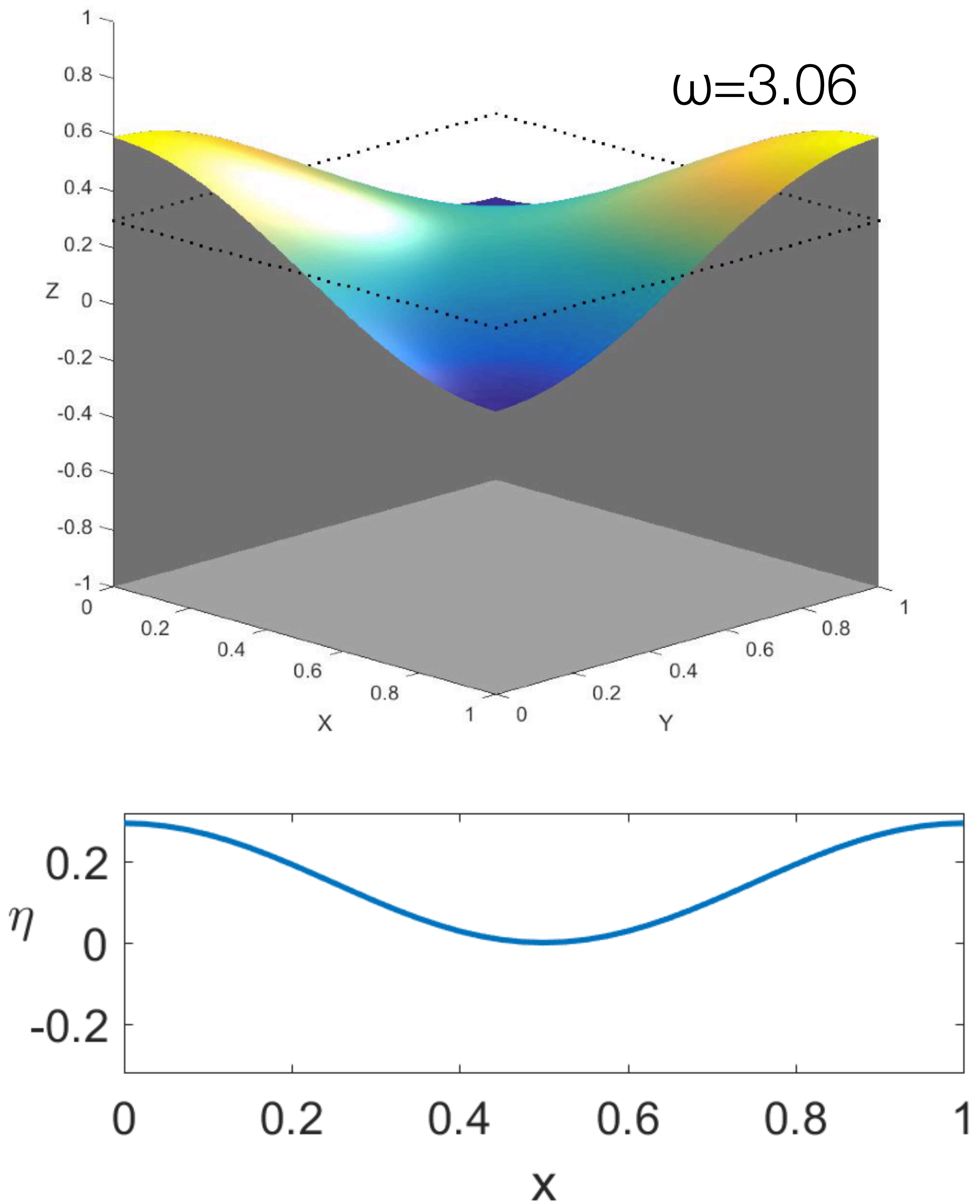


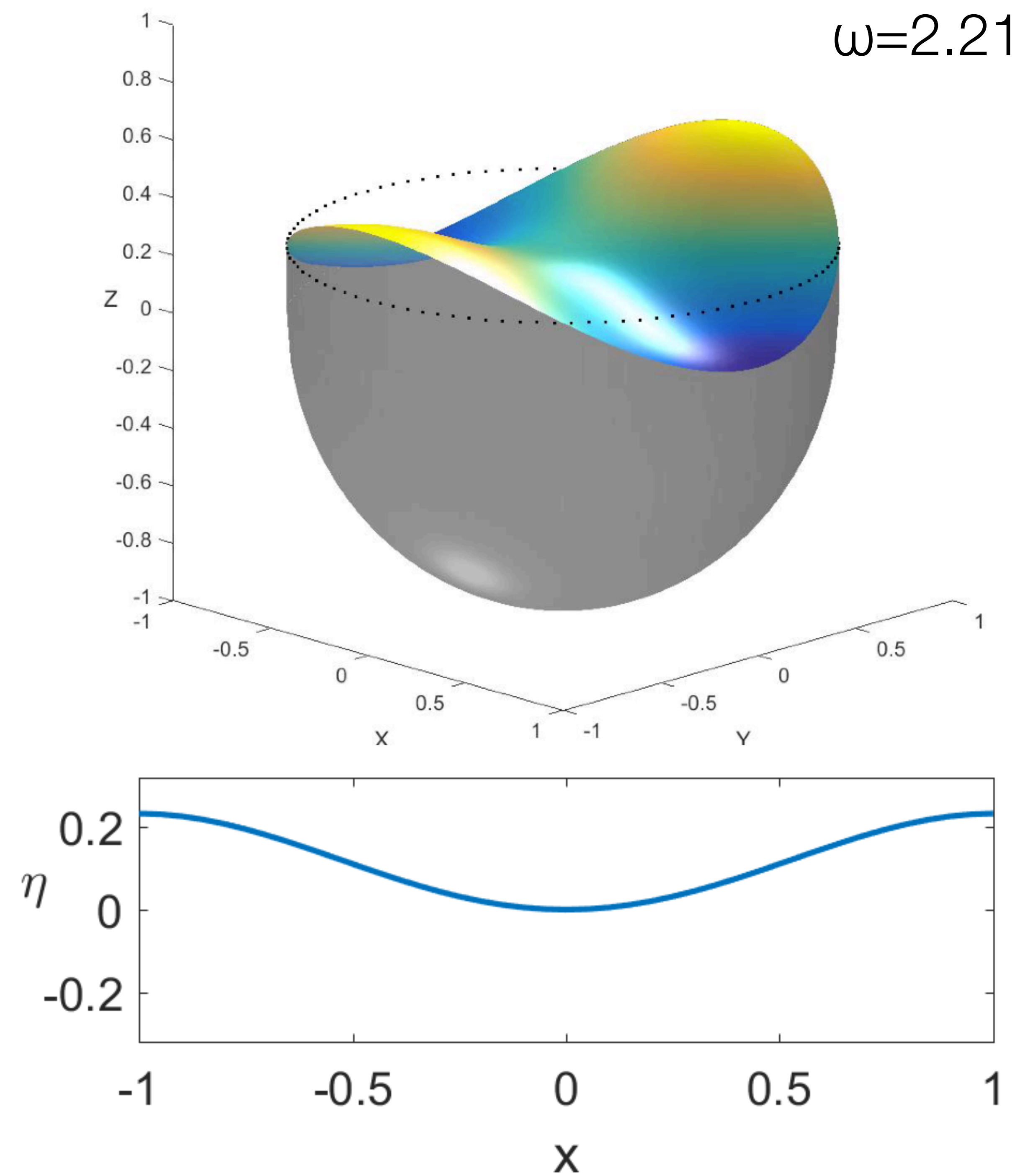
Exact solutions are used to validate the code



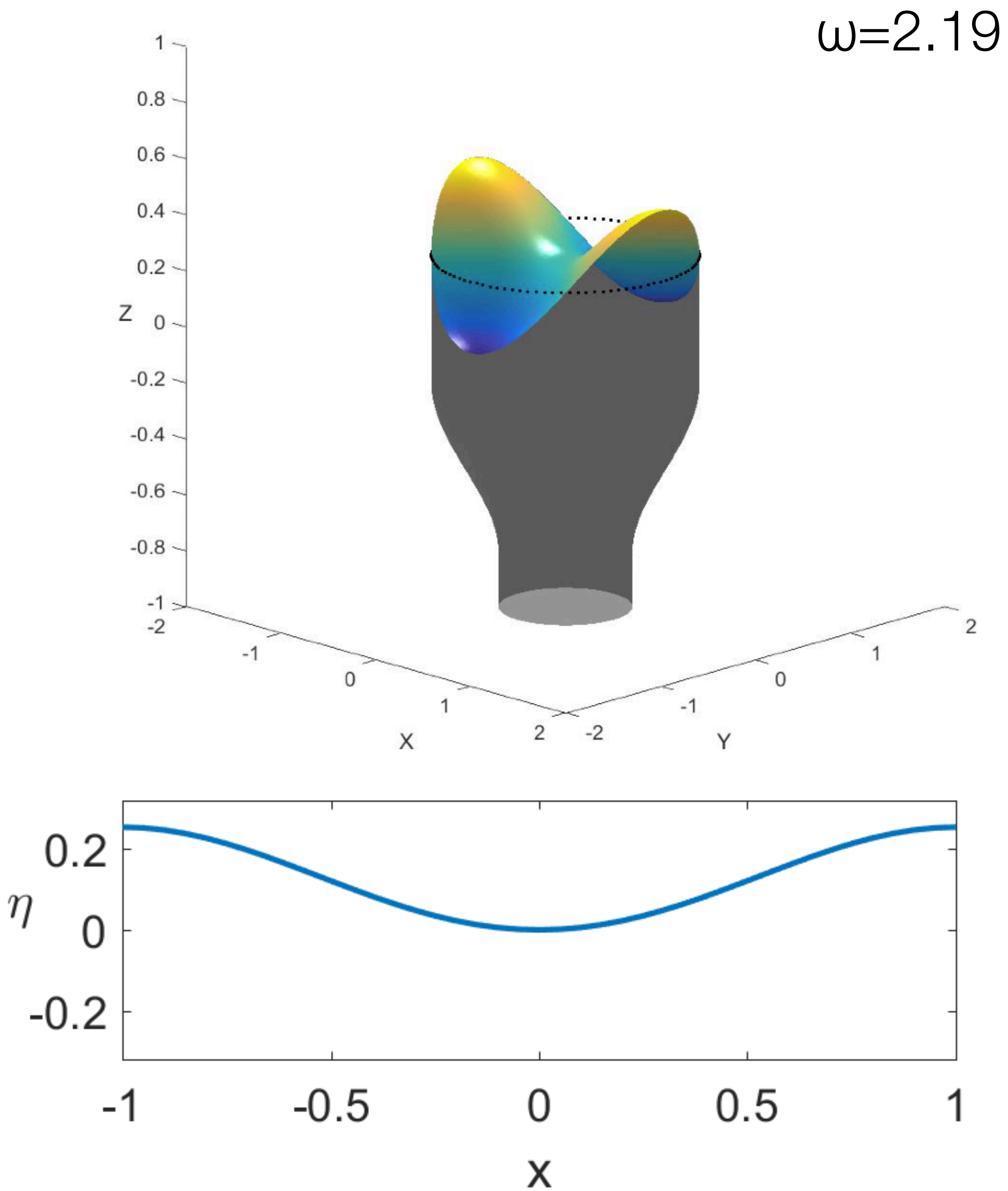


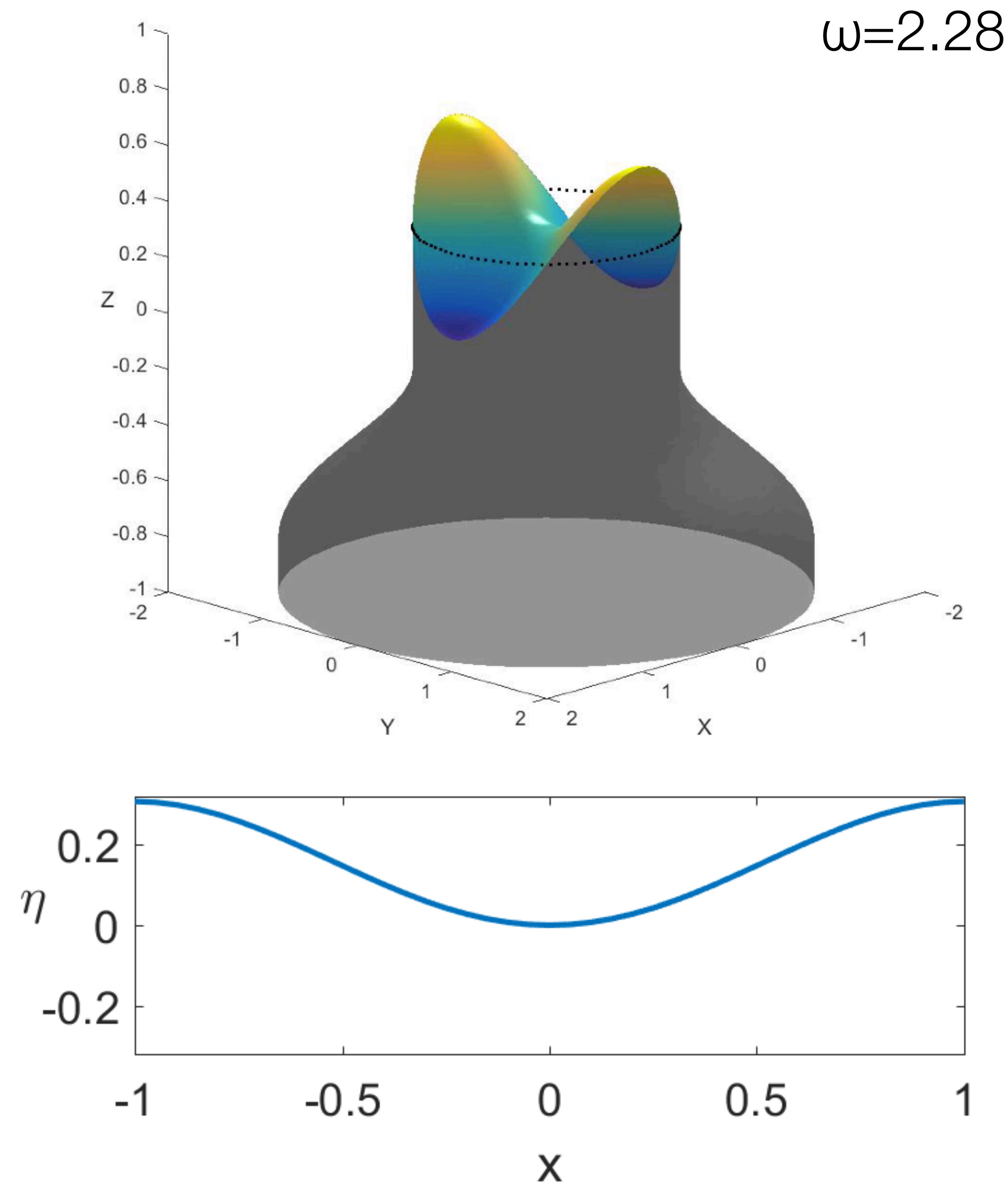
Results: Box and Bowl, Bo=10





Results: Funnel and Vase, Bo=10





Conclusions and Future Directions

- Finish quantifying the effect of surface tension on the shape of the fluid surface for small oscillations.
- Extend the code to finer mesh size. Use Hari Sundar's parallel C code.
- Study implications for NASA SPHERES-Slosh project, which is to collect data in a microgravity environment to improve our understanding of how propellants within rockets behave.

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