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Final exam for MATH 5710/MATH 6880 Optimization Fall 2011

You may use any academic sources.

Return at Monday, December 12, before 2:00 pm by email to cherk@math.utah.edu

1. Apply the conjugate gradient method for minimization of a quadratic function

$$\min_{x_1,x_2,x_3} F(x_1,x_2,x_3), \quad F = x_1^2 + 4x_2^2 + 2x_3^2 + (x_1 + x_3)\, x_2 - x_1\, x_3 - x_1$$

   Initial point is $x_1 = 2,\ x_2 = 0,\ x_3 = 0$. Show that a solution is achieved in a finite number of iterations. Justify the number of iterations.

2. Apply the quasi-Newton method (any modification) to the same problem. Perform one step.

3. Show that a trust region method can be used for minimization of a bell-shape function

$$f(x_1,x_2) = 1 - \frac{1}{1 + x_1^2 + 4x_2^2 + x_1 x_2},$$

   starting from the point $x_1 = 3,\ x_2 = 2$. Compute the first iteration.

   Is a conjugate gradient method applicable to this problem?

   What minimization methods can be used, if the starting point is $x_1 = 0.1,\ x_2 = 0.1$? Explain.

4. Consider a two-players zero sum game with the payoff matrix

$$P = \begin{pmatrix} 0 & 4 & 3 \\ 4 & 1 & 3 \end{pmatrix}.$$

   Find optimal mixed strategies for both players, applying simplex method for the corresponding linear program.

5. Find duals for the following optimization problems

   a) Linear programming:

$$\min_{x_1,x_2,x_3} (x_1 - 2x_2 + x_3)$$

   subject to

$$x_1 + x_2 = 1, \quad 2x_3 + x_1 \geq 2, \quad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

   b) Quadratic programming:

$$\min_{x_1,x_2,x_3} (3x_1^2 + x_2^2 + x_1x_3 + x_3^2 - x_1x_2 - x_2)$$

   subject to

$$x_1 - x_2 \geq 1, \quad x_3 + x_1 = 2, \quad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

6. (i) For both problems in No 5, find KKT optimality conditions. Suggest an algorithm for the solutions. Solve.

   (ii) Apply an interior points method to the problems in No 5: Modify the KKT conditions, derive and explain an algorithm.

7. Consider the problem

$$\min_{x_1,x_2} (x_1^4 + x_2^2 + x_1\, x_2 - x_2)$$

   subject to constraint $x_1 = x_2^2 - 1$. Applying the augmented Lagrangian method, write an algorithm for iterative solution and updating the Lagrange multiplier.

8. Plot the 3d graph of a multimodal function

$$f = 3\sqrt{x_1^2 + x_2^2} + \log(1+x^4+4y^4)\left(1.25 + \cos(4x_1^2 + x_2^3)\right), \quad |x_1| \leq 5,\ |x_2| \leq 5.$$

   What methods can be used for finding its minimum? Suggest an algorithm and discuss.