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**Final exam for  
MATH 5710/MATH 6880 Optimization  
Fall 2010**

You may use any academic sources.

Return at Thursday, December 16, at 9:40.

1. Apply the conjugate gradient method for minimization of a quadratic function

$$\min_{x_1, x_2, x_3} F(x_1, x_2, x_3), \quad F = x_1^2 + x_2^2 + x_3^2 + (x_1 + x_3)x_2 - x_1x_3 - x_1$$

Initial point is  $x_1 = 2$ ,  $x_2 = 0$ ,  $x_3 = 0$ . Show that a solution is achieved in a finite number of iterations. Justify the number of iterations.

2. Apply the quasi-Newton method (any modification) to the same problem. Perform one step.
3. Show that a trust region method can be used for minimization of a bell-shape function

$$f = 1 - \frac{1}{1 + x_1^2 + x_2^2},$$

starting from the point  $x_1 = 2$ ,  $x_2 = 2$ . Compute the first iteration.

Is a conjugate gradient method applicable to this problem?

What minimization methods can be used, if the starting point is  $x_1 = 0.3$ ,  $x_2 = 0.1$ ? Explain.

4. Consider a two-players zero sum game with the payoff matrix

$$M = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 2 & 3 \end{pmatrix}.$$

Find optimal mixed strategies for both players, applying simplex method for the corresponding linear program.

5. Find duals for the following optimization problems

a) Linear programming:

$$\min_{x_1, x_2, x_3} (x_1 + x_2 - 3x_3)$$

subject to

$$x_1 - x_2 = 1, \quad x_3 + x_1 \geq 2, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

b) Quadratic programming:

$$\min_{x_1, x_2, x_3} (x_1^2 + x_2^2 + x_1x_3 + x_3^2 - x_1x_2)$$

subject to

$$x_1 - x_2 \geq 1, \quad x_3 + x_1 = 2, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

6. (i) For both problems in No 5, find KKT optimality conditions. Suggest an algorithm for the solutions. Solve.

(ii) Apply an interior points method to the problems in No 5: Modify the KKT conditions, derive and explain an algorithm.

7. Consider the problem

$$\min_{x_1, x_2} (x_1^4 + x_2^2 + 2x_1x_2 - x_2)$$

subject to constraint  $x_1 = x_2^2 - 1$ . Applying the augmented Lagrangian method, write an algorithm for iterative solution and updating the Lagrange multiplier.

8. Plot the 3d graph of a multimodal function

$$f = 3\sqrt{x_1^2 + x_2^2} + \log(1 + x^4 + 4y^2) (1.3 + \cos(x_1^2 - x_2^3)), \quad |x_1| \leq 5, \quad |x_2| \leq 5.$$

What methods can be used for finding its minimum? Suggest an algorithm and discuss.