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Final exam for MATH 5710/MATH 6880 Optimization Fall 2009

You may use any academic sources.

Return at Friday, December 11, at 9:40.

1. Apply the conjugate gradient method for minimization of a quadratic function

$$\min_{x_1,x_2,x_3} F(x_1,x_2,x_3), \quad F = x_1^2 + x_2^2 + x_3^2 + x_1\,x_2 + x_2\,x_3 + x_3\,x_1$$

   Initial point is $x_1 = 2,\ x_2 = 0,\ x_3 = 0$. Show that a solution is achieved in a finite number of iterations. Justify the number of iterations.

2. Show that a trust region method can be used for minimization of a bell-shape function

$$f = 1 - \frac{1}{1 + x_1^2 + 4\,x_2^2},$$

   starting from the point $x_1 = 2,\ x_2 = 2$. Compute the first iteration. Is a conjugate gradient method applicable to this problem?

   What minimization methods can be used, if the starting point is $x_1 = 0.3,\ x_2 = 0.1$? Explain.

3. Consider a two-players zero sum game with the payoff matrix

$$M = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 1 & 3 \end{pmatrix}.$$

   Find an optimal mixed strategy for both players, applying simplex method for the corresponding linear program. Solve the pair of dual problems.

4. Find duals for the following optimization problems

   a) Linear programming:

$$\min_{x_1,x_2,x_3} (x_1 + x_2 - 3x_3)$$

   subject to

$$x_1 - x_2 = 1, \quad x_3 + x_1 \geq 2, \quad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

   b) Quadratic programming:

$$\min_{x_1,x_2,x_3} (x_1^2 + x_2^2 + x_1 x_3 + x_3^2 - x_1 x_2)$$

   subject to

$$x_1 - x_2 \geq 1, \quad x_3 + x_1 = 2, \quad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

5. (i) For both problems in No 4, find KKT optimality conditions. Suggest an algorithm for the solutions. Solve.

   (ii) Apply an interior points method to the problems in No 4: Modify the KKT conditions, derive and explain an algorithm.

6. Consider the problem

$$\min_{x_1,x_2} (x_1^4 + x_2^2 + x_1\, x_2 - x_2)$$

   subject to constraint $x_1 = x_2^2 - 1$. Applying the augmented Lagrangian method, write an algorithm for iterative solution and updating the Lagrange multiplier.

7. Plot the 3d graph of a multimodal function

$$f = \sqrt{x_1^2 + x_2^2} + \frac{1}{1.3 + \cos(x_1^2 - x_2^2)}, \quad |x_1| \leq 5,\ |x_2| \leq 5.$$

   What methods can be used for finding its minimum? Suggest an algorithm and discuss.