Introduction to Optimization
Math 5770-001, Math 6640-001, ME EN 6025-001
Midterm exam
Search Methods for Unconstrained Optimization

Your name and class number .....

Due Day: Thursday, October 17.

Test the following methods

1. Conjugate gradient (5.1) or, for nonquadratic functions, (5.2)
2. Quasi-Newton BFGS (6.1)
3. Quasi-Newton BFGS (6.2)
4. Nedler-Mead (9.5)

Write the programs using the discussed algorithms. For non-quadratic functions, use any line search method (see Chapter 4), explain. Find minima of the following functions

I. Quadratic A

$$Q_1(x) = x^T A x - b^T x, \quad A = \begin{pmatrix} 4 & 2 & 2 & 4 \\ 2 & 6 & 10 & -1 \\ 2 & 10 & 40 & 3 \\ 4 & -1 & 3 & 10 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 3 \end{pmatrix} \tag{1}$$

Starting point $x_0 = (0, 0, -1, 3)$ Compute minimum of Q_1 by Maple or Matlab and plot distance from the minimum to the approximations at five iterations. Discuss.

Quadratic B

$$Q_2(x) = \sum_{i=1}^{6} \sum_{j=1}^{6} x_i x_j + \sum_{i=1}^{6} x_i^2 - \sum_{i=1}^{6} \left(\frac{x_i}{i} \right) \tag{2}$$

Starting point $x_0 = 0$ Compute minimum of Q_2 by Maple or Matlab and plot distance from the minimum to the approximations at seven iterations. Discuss.

II. Rosenblock function (see problem 2.1)

$$F_R(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{3}$$

Starting point $x_0 = (0, 0)$. Plot 20 iterations in the x_1, x_2 plane. Discuss.

III. Exotic functions:
a. Flat surface:

$$F_a(x) = \exp\left(-x_1^2 - x_2^2\right) \tag{4}$$

Two starting points $x_0 = (1, 1)$ and $x_0 = (-1, 1)$. Plot 30 iterations in the x_1, x_2 plane. Discuss.

b. Small Hessian

$$F_b(x) = \sqrt{0.2 + x_1^2 + 2x_2^2} \tag{5}$$

Starting point $x_0 = (2, 2)$. Plot 20 iterations in the x_1, x_2 plane. Discuss.

Note: At Tuesday, October 15, the class is cancelled due to the SES Annual Technical Meeting.