Final exam.
MATH 5770-001 and 6640-001 and ME EN 6025-001
Introduction to Optimization

Due Day: Friday, December 13, 2019

1. Using Lagrange multipliers method, solve the problem

$$\min_{x_1,\dots,x_n} \sum_{i=1}^{n} a_i^2 x_i \quad \text{subject to} \quad \sum_{i=1}^{n} \frac{c}{x_i - b} = 1$$

   assuming that $x_i > b$. Here, a_i, b, c are real parameters.

2. Using Lagrange Multiplyers method, solve the problem

$$\min_{x_1,x_2} (x_1 - 1)^2 + (x_2 - t)^2, \quad \text{subject to} \quad |x_1| + |x_2| \leq 2$$

   Here t is a real parameter. Find the solutions for all values of t. Sketch the graph of the feasible region and illustrate the found solutions.

3. Convert the following problem to the standard LP form

$$\min_{x_1,\dots,x_4} x_1 + x_2 + 3x_3 + x_4$$

   subject to

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + 3x_3 &\leq 1 \\ x_2 + x_4 &\leq 10 \\ 1 \leq x_2 &\leq 3 \\ -1 \leq x_3 &\leq 1 \end{aligned}$$

4. Solve
$$\min_{x\in R_n} (x-x_0)^T(x-x_0) \quad \text{subject to} \quad Ax = b$$
Here, $x \in R_n,\ b \in R_m,\ n > m$. Rank $A = m$ (in other words, all constraints are linearly independent.

   Find the formulas for the vector Lagrange multiplyer, and the optimal value x_* of x.

5. Consider the problem
$$\min_{x_1,x_2} e^{x_1+x_2} \quad \text{subject to} \quad x_1^2 + x_2^2 \leq 1 \tag{1}$$
Write down the KKT conditions.

   Formulate the problem using logarithmic barrier function [formula (19.4) in the textbok] for accounting for the constraint. Write down the KKT conditions for the obtained Interior-Point problem. Compute the minimum, using different values of the barrier parameter $\mu = 1, 01, 0.001$. Comment.

6. Use quadratic penalty to solve problem in eq.(1). Compute the minimum, using different values of the penalty parameter $\mu = 0.1, 1, 10, 30$. Comment.

7. Use augmented Lagrangian method to numerically solve problem
$$\min_{x_1,x_2,x_3} (x_1-3)^2 + 2x_2x_3 + x_3^2 \quad \text{subject to} \quad x_1^2 + x_2^2 + x_3^2 = 4$$
Starting at the point $x^{(0)} = [2, 0, 0]$. Perform 8 iterations. Comment.