HW 5

1. Find conjugate $f^*(x^*)$ to the function

$$f(x) = \frac{1}{2} a\, x^2 + b\, x + \frac{1}{2} c$$

   Compare $\min f(x)$ and $\max f^*(x^*)$.

   Compute $x\, x^*$

2. Find conjugate $f^*(x^*)$ to the function

$$f(x) = \max \left\{ \frac{(x-1)^2}{2}, \frac{(x-1)^2}{2} \right\}$$

   Using this result, compute conjugate $g^*(x^*)$ to the function

$$g(x) = \min \left\{ \frac{(x-1)^2}{2}, \frac{(x-1)^2}{2} \right\}$$

   Compute the convex envelope of $g(x)$.

3. Using Ritz method, approximate solution to the problem

$$I = \min_{u(x)} \int_0^1 \left(x(u')^2 + (1-x^2)u^2 \right) dx, \quad u(0) = 1,\ u(1) = 0$$

   Represent the solution as a sum of polynoms and trigonometric functions and account for boundary conditions:

$$u(x) = (1-x) + x(1-x) \left(\sum_{k=0}^{N} (a_{2k} \cos(\pi k x) + a_{2k+1} \sin(\pi k x), ,) \right)$$

   find optimal valuers of parameters, graph the solution. How the graph varies when N increases?

4. Using Galerkin method, approximate the solution to the ODE

$$u''' - xu'' + 1 - x)^2 u' - x(1-x)u + x^5 = 0, \quad u(0) = 1, u(1) = u'(1) = 0$$

Use polynomials up to degree N with unknown coefficients to approximate the solution and to account for boundary conditions, use polynomials as the trial functions ψ. Graph the approximates of the solution for different values N.

Refs:

1. http://fischerp.cs.illinois.edu/tam470/refs/galerkin2.pdf
2. http://www.sd.ruhr-uni-bochum.de/downloads/Galerkin_method.pdf
3. https://en.wikipedia.org/wiki/Galerkin_method