

## HW1

1. Derive the Euler equation. Solve

$$\min_{u(x), b} \int_0^b ((u')^2 + 2xu) dx \quad u(0) = 0, \quad u(b) = 0,$$

2. Derive the Euler equation and boundary conditions. Solve. Find the first integral

$$\min_{u(x)} \left[ \int_0^1 ((u')^2 - \omega^2 u^2) dx + 2u(1), \right] \quad u(0) = 0,$$

3. Derive the Euler equation and boundary conditions. Solve. Explain the dependence of the solution on  $a$ .

$$\min_{u(x)} \int_0^b ((u')^2 + a u u' + u) dx \quad u(0) = 0.$$

4. Derive the Euler equation and boundary conditions. Solve, using Maple

$$\min_{u(x)} \int_0^b ((u'')^2 - a (u')^2 + b u) dx \quad u(0) = 0, \quad u'(0) = 0$$