HW 4

5500-2018

1. Describe potential and kinetic energy of the following two-mass system: Mass m_1 is attached by a linear spring of the length L with the spring constant C to steady support, mass M is attached by the similar spring to the mass m.

   Write the kinetic and potential energy of the system, find first integrals. Find the motion of both masses. Then consider the case when $m \to 0$.

   Consider two cases:
   (a) Initially, the first spring is elongated; the elongation is equal to Δ. Initial velocities are equal to zero.
   (b) Initially, both springs are elongated; the elongations are equal to $\frac{1}{2}\Delta$. Initial velocities are equal to zero.

   Compare the energy in both cases. Comment on the difference in the solutions.

2. Necessary conditions.
   Consider the Lagrangian

$$L = \frac{1}{2} m\, x\, \dot{x}^2 - x(1+x), \quad x \in [0,\ A), \quad A >> 1$$

   Write the equation for the stationary trajectory. Does it correspond to the minimum? Check Legendre and nonlocal Jacobi-type condition.

3. Filtration (linear)
   The received signal $h(t)$ is the sum of the true response $s(t)$ and the noice $v(t)$

$$s(t) = t(L-t), \quad v(t) = 2\sin(10\pi L t) + \cos(7\pi L t), \quad t \in [0, L], \quad L = 2$$

   Filter the signal using the penalties $p_1 = \alpha(u')^2$ and $p_2 = \alpha(u'')^2$.

   Graph the received and filtered signals for several values of α. Discuss.