5500-18 HW 2

1. **"Inverse brachistochrone":** Assume that the fastest trajectory is a part of a circle
$$y(x) = \sqrt{A^2 - x^2}$$
Find the speed dependence $v(y)$.

2. The astronauts arrives at planet EasterEgg with a viscous atmosphere and find that the speed of travel depends on the distance r from the center as $v(r)$. They need to find out the brachistochrone trajectory to travel form point (r_1, θ_1) to point (r_2, θ_2).

   Derive the equation of brachistochrone in polar coordinates.

   Find the trajectory, if $v(r) = r$.

3. Find the boundary conditions for the point in which the brachistochrone that started from the point $(0, 0)$ meets the circle $x^2 + y^2 = R^2$.

4. The thermal equilibrium in a bar $0 \leq x \leq 1$ is described by the boundary value problem for the temperature $T(x)$
$$T'' = \gamma(x) \in (0 \leq x \leq 1), \quad T' + \alpha(T - T_0)^4 = 0 \text{ at } x = 1, \ T' = 0 \text{ at } x = 1.$$
   where γ is the density of heat sources, α is the radiation constant, T_0 is a constant outside temperature.

   Write the variational problem which minimizer describes the equilibrium.