

HW 5

Hamiltonian and Legendre Transform

1. Consider a plane double pendulum:
Mass m_1 is hanged on a rod of the length l_1 that is attached to the base point with coordinated $(0, 0)$. Mass m_2 is hanged on a rod of the length l_2 that is attached to mass m_1 .
Derive the kinetic and potential energy of the system via angles between the rods and the vertical, find Lagrangian and Hamiltonian, find first integrals, write the differential equation of motion of the masses.
2. Find the Legendre transform of the functions:

$$f_1(x) = -\log(x), \quad f_2(x) = \frac{1}{2}ax^2 + bx + c$$

$$f_3(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2) + x_1x_2$$

Find the Legendre transform of the obtained transforms (second transform).