HW 4 for Calculus of Variations 2012

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1. Compute convex envelopes of the functions

$$f_1(u) = \min\left\{(u-1)^2, (u+1)^2\right\}$$

$$f_2(u) = -\frac{1}{1+u^2}$$

$$f_3(u_1, u_2) = \begin{cases} 0 & \text{if } u_1^2 + u_2^2 = 0 \\ 1 + u_1^2 + u_2^2 & \text{otherwise} \end{cases}$$

2. Consider variational problems

$$\inf \int_0^1 (u^2 + f_i(u'))dx, \quad u(0) = a, i = 1, 2$$

Find a relaxed formulation, and write the equations for the minimizers.

3. Solve the equation for eikonal $S = S(x_1, x_2)$

$$(\nabla S)^2 = 1, \quad |x_1| + |x_2| > 1, \quad S = 0 \text{ if } |x_1| + |x_2| = 1$$

(Guess the solution, assuming that $|x_1| + |x_2| = 1$ is the radiating curve, than check that the solution is correct.)