Calculus of Variations
HW 2

Andrej Cherkaev

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1. Two masses m_1 and m_2 are sequentially jointed to a wall ($x = 0$) by two springs. At equilibrium, the distance from the wall is equal to one for the first mass, and equal to two for the second one. The kinetic energy K_i of each mass is $K_i = \frac{1}{2} m_i \dot{x}_i^2$, the potential energy of the first and second springs are

$$\Pi_1 = \frac{C}{2}(x_1 - 1)^2, \quad \Pi_2 = \frac{C}{2}(x_2 - x_1 - 1)^2$$

where $C > 0$ is a spring constant.

a. Compose the variational problem of minimal action of the whole system, find the Euler-Lagrange equations for the motion.

b. Numerically integrate the equations for the following parameters

$$C = 1, \ m_1 = 0.1, \ m_2 = 1$$

with initial conditions $x_1 = 1.2, \ x_2 = 2.2, \ \dot{x}_1 = 0, \ \dot{x}_2 = 0$. Graph the motion of the first and second mass.

c. Is it possible to approximate the motion of m_2 with one-mass system? Write the equations, compare the energy of the two systems.

2. Assume that the speed of motion in a x, y-plane depends on y coordinate as $v_1 = e^y$, $v_2 = \frac{1}{1+y^2}$.

a. Find the fastest path between points $(0, 0)$ and $(1, 1)$. Use Maple, if needed.

b. Compute the Hamiltonian.

3. Perform the Legendre and Weierstrass tests for the Lagrangian

$$F = [(u')^2 - u^2]^2$$