

Homework for math 5500: Calculus of Variations Spring 2012

Andrej Cherkaev

1 HW1

1. Consider

$$J_1 = \min_{u(x):u(0)=1} \int_0^1 (a(u')^2 + 2b u u' + c u^2) dx, \quad a \geq 0, \quad c \geq 0$$

Find Euler equation and the natural boundary condition at $x = 1$. Solve. Discuss dependence on b . What happens when $a = c = 0$?

2. Approximate $y = \text{sign } x$, $x \in [-1, 1]$ by a function $u(x)$ using quadratic penalty, that is minimize

$$J_2 = \min_{u(x)} \frac{1}{2} \int_{-1}^1 [(u - y)^2 + \varepsilon^2 (u')^2] dx.$$

Here

$$\text{sign } x = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$

3. Approximate $y = \sin x$, $x \in [0, 2\pi]$ by a function $u(x)$ using TV penalty, that is minimize

$$J_3 = \min_{u(x)} \int_0^{2\pi} \left[\frac{1}{2} (y - u)^2 + \varepsilon |u'| \right] dx$$