

**MATH 5740/MATH 6870 001**  
**MATH MODELING 2010**  
**09:40 AM-10:30 AM JWB 208**

Introduction

# Models and reality

- *Theory attracts practice as the magnet attracts iron.*  
*Gauss*
- We live in the world of models:
- Great models: Universe, Evolution, Social organization  
– determine our life forcing our judgment, decisions,  
and feelings
- Paradigms arrive in the form of new models: Examples:  
Neo-Darwinism, fractals, democracy, solitons.
- Models as reference points
- ***show fractal website***

**Galilei, Galileo (1564 - 1642):**

[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.

*Opere Il Saggiatore* p. 171.

# Definition from Wiki

- A **mathematical model** uses [mathematical](#) language to describe a [system](#).
- Mathematical models are used in the [natural sciences](#) and [engineering](#) disciplines (such as [physics](#), [biology](#), [earth science](#), [meteorology](#), and [engineering](#)) and in the [social sciences](#) (such as [economics](#), [psychology](#), [sociology](#) and [political science](#))..
- The process of developing a mathematical model is termed 'mathematical modelling' (also modeling).

# Model is an intentionally distorted system description that emphasizes desirable features

*Everything should be made as simple as possible, but not simpler. A. Einstein*

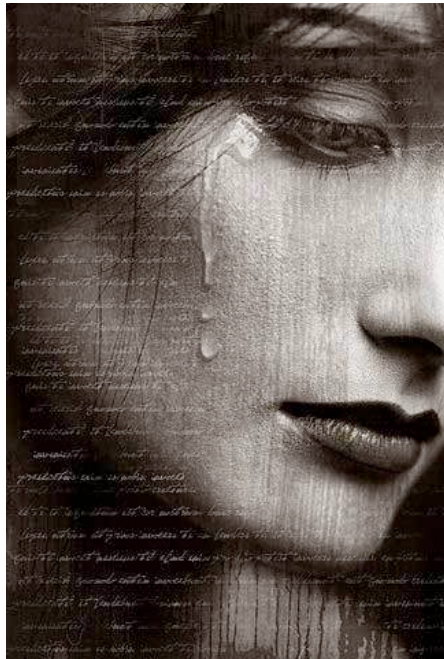
⇒ Model is not unique

- Bridge: for traffic model – a piece of road
- For flight navigation model – an obstacle
- For vibration model – an unbreakable elastic structure
- For strength model – an elastic-plastic structure
- For bungee jumping -- a base.
- For an artist – a shape.



The Millau Viaduct

# Three visual models of the same emotion



# The use of math models

**Goals:** descriptive or design/optimization (natural or engineering model).

**Range of applicability** Ideal gas, Black matter, optimization

**Validation:** Mental experiment vs. real experiment. Galileo.

**Results:** Numerical or analytic. Simulation models.

# Types of math models, by math methods

- Curve fitting vs. equation solving  
(based on a priori principles or empirical)
- Static vs. dynamic
- Continuum vs. discrete
- Deterministic vs. stochastic
- Game theoretical vs. probabilistic



# Types of models, by the level of abstraction

- geometric model – from Plato bodies to fractals
- curve fitting, statistics
- differential equations
- variational principles

example:

Copernicus, Tycho Brahe, Kepler, Newton, Lagrange

## **Ingredients:**

Modeling involves natural/social sciences (physics-biology-geology- sociology-etc), engineering, and mathematical techniques

## **Mathematical tools**

Geometry, Equation solving, ODE, PDE, numerics, game theory, probability, statistics, optimization and control theory,

## **Creativity**

# Modeling of the Universe (Problem 1)

Write a short essay (< 2 pages) about development of the model of Universe by

- Ptolemy (Klaúdios Ptolemaîos),
- Nicolaus *Copernicus*
- Tycho Brahe,
- Johannes *Kepler*,
- Isaac Newton,
- Albert Einstein.

Characterize the models as empirical, or data fitting, or equation solving, or a general-principle-based, and comment on complexity of the models and motivation for the improvement/development.

Use Internet, Wiki, or any other available sources

<http://astro.unl.edu/naap/ssm/animations/ptolemaic.swf>

# Class Outline (wish list)

- Introduction. Types of Models.
- Great Models: Copernicus, Newton, Einstein. Black holes. Fractals.
- Population dynamics. Simple models and their combinations
- Epidemics: disease spread
- Dimensionality analysis (Parachute problem).
- Discrete and continuum waves: domino train and traffic wave
- Model of diffusion. ????
- Optimal design -- a friction stopper
- Thresholds: model of damage propagation
- Stochastic Modeling for uncertainties
- Game theory: Modeling for the worse case scenario

**Suggestions are welcome!**

# Organization of the class work

- For the project work the class will be divided into groups of three
- A presenter will orally present the group's work, the report will be written. The group get a grade for the project.
- The groups will be formed for each project, the roles will be reassigned so that each student will be researcher and presenter.
- **Problem 2. (one week)**
- Write an algorithm for dividing the class into groups of three for each project. Next time, bring in your ideas.