CHAPTER 1

Mathematical Modeling and Engineering Problem Solving

Knowledge and understanding are prerequisites for the effective implementation of any tool. No matter how impressive your tool chest, you will be hard-pressed to repair a car if you do not understand how it works.

This is particularly true when using computers to solve engineering problems. Although they have great potential utility, computers are practically useless without a fundamental understanding of how engineering systems work.

This understanding is initially gained by empirical means—that is, by observation and experiment. However, while such empirically derived information is essential, it is only half the story. Over years and years of observation and experiment, engineers and scientists have noticed that certain aspects of their empirical studies occur repeatedly. Such general behavior can then be expressed as fundamental laws that essentially embody the cumulative wisdom of past experience. Thus, most engineering problem solving employs the two-pronged approach of empiricism and theoretical analysis (Fig. 1.1).

It must be stressed that the two prongs are closely coupled. As new measurements are taken, the generalizations may be modified or new ones developed. Similarly, the generalizations can have a strong influence on the experiments and observations. In particular, generalizations can serve as organizing principles that can be employed to synthesize observations and experimental results into a coherent and comprehensive framework from which conclusions can be drawn. From an engineering problem-solving perspective, such a framework is most useful when it is expressed in the form of a mathematical model.

The primary objective of this chapter is to introduce you to mathematical modeling and its role in engineering problem solving. We will also illustrate how numerical methods figure in the process.

1.1 A SIMPLE MATHEMATICAL MODEL

A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. In a very general sense, it can be represented as a functional relationship of the form

$$\begin{matrix}\text{Dependent}\\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent}\\ \text{variables}\end{matrix}, \text{parameters}, \begin{matrix}\text{forcing}\\ \text{functions}\end{matrix}\right) \tag{1.1}$$

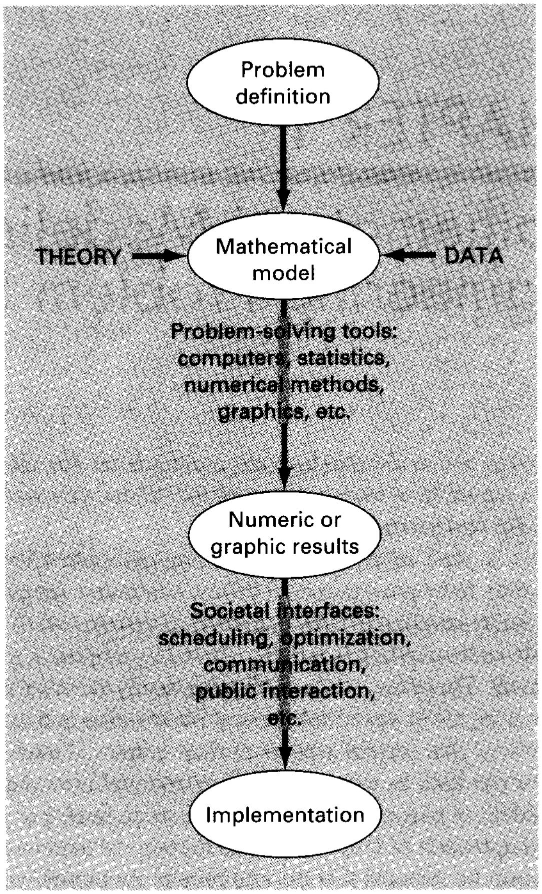


FIGURE 1.1
The engineering problem-solving process.

where the *dependent variable* is a characteristic that usually reflects the behavior or state of the system; the *independent variables* are usually dimensions, such as time and space, along which the system's behavior is being determined; the *parameters* are reflective of the system's properties or composition; and the *forcing functions* are external influences acting upon it.

The actual mathematical expression of Eq. (1.1) can range from a simple algebraic relationship to large complicated sets of differential equations. For example, on the basis of his observations, Newton formulated his second law of motion, which states that the time rate of change of momentum of a body is equal to the resultant force acting on it. The mathematical expression, or model, of the second law is the well-known equation

$$F = ma \tag{1.2}$$

where F = net force acting on the body (N, or kg m/s^2), m = mass of the object (kg), and a = its acceleration (m/s^2).

FIGURE 1.2
Schematic diagram of the forces acting on a falling parachutist. F_D is the downward force due to gravity. F_U is the upward force due to air resistance.

The second law can be recast in the format of Eq. (1.1) by merely dividing both sides by m to give

$$a = \frac{F}{m} \tag{1.3}$$

where a = the dependent variable reflecting the system's behavior, F = the forcing function, and m = a parameter representing a property of the system. Note that for this simple case there is no independent variable because we are not yet predicting how acceleration varies in time or space.

Equation (1.3) has several characteristics that are typical of mathematical models of the physical world:

1. It describes a natural process or system in mathematical terms.
2. It represents an idealization and simplification of reality. That is, the model ignores negligible details of the natural process and focuses on its essential manifestations. Thus, the second law does not include the effects of relativity that are of minimal importance when applied to objects and forces that interact on or about the earth's surface at velocities and on scales visible to humans.
3. Finally, it yields reproducible results and, consequently, can be used for predictive purposes. For example, if the force on an object and the mass of an object are known, Eq. (1.3) can be used to compute acceleration.

Because of its simple algebraic form, the solution of Eq. (1.2) can be obtained easily. However, other mathematical models of physical phenomena may be much more complex, and either cannot be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution. To illustrate a more complex model of this kind, Newton's second law can be used to determine the terminal velocity of a free-falling body near the earth's surface. Our falling body will be a parachutist (Fig. 1.2). A model for this case can be derived by expressing the acceleration as the time rate of change of the velocity (dv/dt) and substituting it into Eq. (1.3) to yield

$$\frac{dv}{dt} = \frac{F}{m} \tag{1.4}$$

where v is velocity (m/s). Thus, the mass multiplied by the rate of change of the velocity is equal to the net force acting on the body. If the net force is positive, the object will accelerate. If it is negative, the object will decelerate. If the net force is zero, the object's velocity will remain at a constant level.

Next, we will express the net force in terms of measurable variables and parameters. For a body falling within the vicinity of the earth (Fig. 1.2), the net force is composed of two opposing forces: the downward pull of gravity F_D and the upward force of air resistance F_U:

$$F = F_D + F_U \tag{1.5}$$

If the downward force is assigned a positive sign, the second law can be used to formulate the force due to gravity, as

$$F_D = mg \tag{1.6}$$

where g = the gravitational constant, or the acceleration due to gravity, which is approximately equal to 9.8 m/s^2.

The velocity of a falling body

$$m\frac{dv}{dt} = mg - cv$$

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

$$\frac{dv}{dt} = -\frac{c}{m}\left(-\frac{m}{c}g + v\right)$$

$$\frac{1}{-\frac{m}{c}g + v}dv = -\frac{c}{m}dt$$

$$\int \frac{1}{-\frac{m}{c}g + v}dv = \int\left(-\frac{c}{m}dt\right)$$

$$\ln\left|-\frac{m}{c}g + v\right| = -\frac{c}{m}t + C$$

$$\left|-\frac{m}{c}g + v\right| = e^{-\frac{c}{m}t + C}$$

$$-\frac{m}{c}g + v = e^{C}e^{-\frac{c}{m}t}$$

If $v(0) = 0$, then

$$-\frac{m}{c}g = e^{C}e^{0}, \text{ so } -\frac{m}{c}g = e^{C}$$

So,

$$-\frac{m}{c}g + v = -\frac{m}{c}ge^{-\frac{c}{m}t}$$

$$v = -\frac{m}{c}ge^{-\frac{c}{m}t} + \frac{m}{c}g$$

$$v = \frac{mg}{c}\left(1 - e^{-\frac{c}{m}t}\right)$$

Air resistance can be formulated in a variety of ways. A simple approach is to assume that it is linearly proportional to velocity[1] and acts in an upward direction, as in

$$F_U = -cv \tag{1.7}$$

where c = a proportionality constant called the *drag coefficient* (kg/s). Thus, the greater the fall velocity, the greater the upward force due to air resistance. The parameter c accounts for properties of the falling object, such as shape or surface roughness, that affect air resistance. For the present case, c might be a function of the type of jumpsuit or the orientation used by the parachutist during free-fall.

The net force is the difference between the downward and upward force. Therefore, Eqs. (1.4) through (1.7) can be combined to yield

$$\frac{dv}{dt} = \frac{mg - cv}{m} \tag{1.8}$$

or simplifying the right side,

$$\frac{dv}{dt} = g - \frac{c}{m}v \tag{1.9}$$

Equation (1.9) is a model that relates the acceleration of a falling object to the forces acting on it. It is a *differential equation* because it is written in terms of the differential rate of change (dv/dt) of the variable that we are interested in predicting. However, in contrast to the solution of Newton's second law in Eq. (1.3), the exact solution of Eq. (1.9) for the velocity of the falling parachutist cannot be obtained using simple algebraic manipulation. Rather, more advanced techniques such as those of calculus must be applied to obtain an exact or analytical solution. For example, if the parachutist is initially at rest ($v = 0$ at $t = 0$), calculus can be used to solve Eq. (1.9) for

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right) \tag{1.10}$$

Note that Eq. (1.10) is cast in the general form of Eq. (1.1), where $v(t)$ = the dependent variable, t = the independent variable, c and m = parameters, and g = the forcing function.

EXAMPLE 1.1 Analytical Solution to the Falling Parachutist Problem

Problem Statement. A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Use Eq. (1.10) to compute velocity prior to opening the chute. The drag coefficient is equal to 12.5 kg/s.

Solution. Inserting the parameters into Eq. (1.10) yields

$$v(t) = \frac{9.8(68.1)}{12.5}\left(1 - e^{-(12.5/68.1)t}\right) = 53.39\left(1 - e^{-0.18355t}\right)$$

which can be used to compute

[1] In fact, the relationship is actually nonlinear and might better be represented by a power relationship such as $F_U = -cv^2$. We will explore how such nonlinearities affect the model in a problem at the end of this chapter.

t, s	v, m/s
0	0.00
2	16.40
4	27.77
6	35.64
8	41.10
10	44.87
12	47.49
∞	53.39

According to the model, the parachutist accelerates rapidly (Fig. 1.3). A velocity of 44.87 m/s (100.4 mi/h) is attained after 10 s. Note also that after a sufficiently long time, a constant velocity, called the *terminal velocity,* of 53.39 m/s (119.4 mi/h) is reached. This velocity is constant because, eventually, the force of gravity will be in balance with the air resistance. Thus, the net force is zero and acceleration has ceased.

Equation (1.10) is called an *analytical,* or *exact, solution* because it exactly satisfies the original differential equation. Unfortunately, there are many mathematical models that cannot be solved exactly. In many of these cases, the only alternative is to develop a numerical solution that approximates the exact solution.

As mentioned previously, *numerical methods* are those in which the mathematical problem is reformulated so it can be solved by arithmetic operations. This can be illustrated

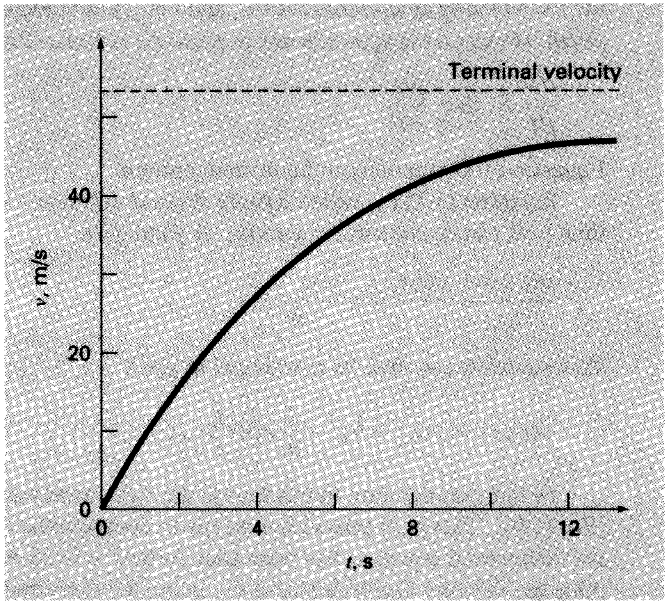


FIGURE 1.3
The analytical solution to the falling parachutist problem as computed in Example 1.1. Velocity increases with time and asymptotically approaches a terminal velocity.

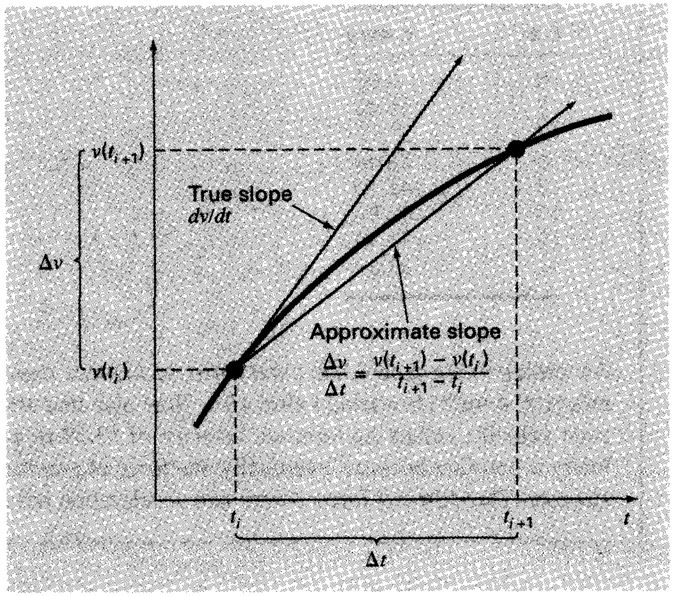


FIGURE 1.4
The use of a finite difference to approximate the first derivative of v with respect to t.

for Newton's second law by realizing that the time rate of change of velocity can be approximated by (Fig. 1.4):

$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} \tag{1.11}$$

where Δv and Δt = differences in velocity and time computed over finite intervals, $v(t_i)$ = velocity at an initial time t_i, and $v(t_{i+1})$ = velocity at some later time t_{i+1}. Note that $dv/dt \cong \Delta v/\Delta t$ is approximate because Δt is finite. Remember from calculus that

$$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Equation (1.11) represents the reverse process.

Equation (1.11) is called a *finite divided difference* approximation of the derivative at time t_i. It can be substituted into Eq. (1.9) to give

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v(t_i)$$

This equation can then be rearranged to yield

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i)\right](t_{i+1} - t_i) \tag{1.12}$$

Notice that the term in brackets is the right-hand side of the differential equation itself [Eq. (1.9)]. That is, it provides a means to compute the rate of change or slope of v. Thus, the differential equation has been transformed into an equation that can be used to determine the velocity algebraically at t_{i+1} using the slope and previous values of v and t. If you are given an initial value for velocity at some time t_i, you can easily compute velocity at a later time t_{i+1}. This new value of velocity at t_{i+1} can in turn be employed to extend the computation to velocity at t_{i+2} and so on. Thus, at any time along the way,

New value = old value + slope × step size

EXAMPLE 1.2 Numerical Solution to the Falling Parachutist Problem

Problem Statement. Perform the same computation as in Example 1.1 but use Eq. (1.12) to compute velocity. Employ a step size of 2 s for the calculation.

Solution. At the start of the computation ($t_i = 0$), the velocity of the parachutist is zero. Using this information and the parameter values from Example 1.1, Eq. (1.12) can be used to compute velocity at $t_{i+1} = 2$ s:

$$v = 0 + \left[9.8 - \frac{12.5}{68.1}(0)\right]2 = 19.60 \text{ m/s}$$

For the next interval (from $t = 2$ to 4 s), the computation is repeated, with the result

$$v = 19.60 + \left[9.8 - \frac{12.5}{68.1}(19.60)\right]2 = 32.00 \text{ m/s}$$

The calculation is continued in a similar fashion to obtain additional values:

t, s	v, m/s
0	0.00
2	19.60
4	32.00
6	39.85
8	44.82
10	47.97
12	49.96
∞	53.39

The results are plotted in Fig. 1.5 along with the exact solution. It can be seen that the numerical method captures the essential features of the exact solution. However, because we have employed straight-line segments to approximate a continuously curving function, there is some discrepancy between the two results. One way to minimize such discrepancies is to use a smaller step size. For example, applying Eq. (1.12) at 1-s intervals results in a smaller error, as the straight-line segments track closer to the true solution. Using hand calculations, the effort associated with using smaller and smaller step sizes would make such numerical solutions impractical. However, with the aid of the computer, large numbers of calculations can be performed easily. Thus, you can accurately model the velocity of the falling parachutist without having to solve the differential equation exactly.

As in the previous example, a computational price must be paid for a more accurate numerical result. Each halving of the step size to attain more accuracy leads to a doubling

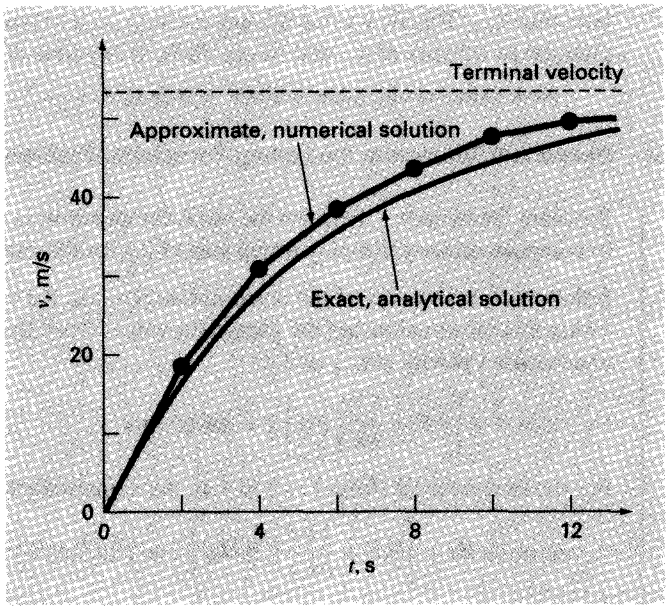


FIGURE 1.5
Comparison of the numerical and analytical solutions for the falling parachutist problem.

of the number of computations. Thus, we see that there is a trade-off between accuracy and computational effort. *Such trade-offs figure prominently in numerical methods and constitute an important theme of this book.* Consequently, we have devoted the Epilogue of Part One to an introduction to more of these trade-offs.

1.2 CONSERVATION LAWS AND ENGINEERING

Aside from Newton's second law, there are other major organizing principles in engineering. Among the most important of these are the conservation laws. Although they form the basis for a variety of complicated and powerful mathematical models, the great conservation laws of science and engineering are conceptually easy to understand. They all boil down to

$$\text{Change} = \text{increases} - \text{decreases} \tag{1.13}$$

This is precisely the format that we employed when using Newton's law to develop a force balance for the falling parachutist [Eq. (1.8)].

Although simple, Eq. (1.13) embodies one of the most fundamental ways in which conservation laws are used in engineering—that is, to predict changes with respect to time. We give Eq. (1.13) the special name *time-variable* (or *transient*) computation.

Aside from predicting changes, another way in which conservation laws are applied is for cases where change is nonexistent. If change is zero, Eq. (1.13) becomes

$$\text{Change} = 0 = \text{increases} - \text{decreases}$$

or

$$\text{Increases} = \text{decreases} \tag{1.14}$$

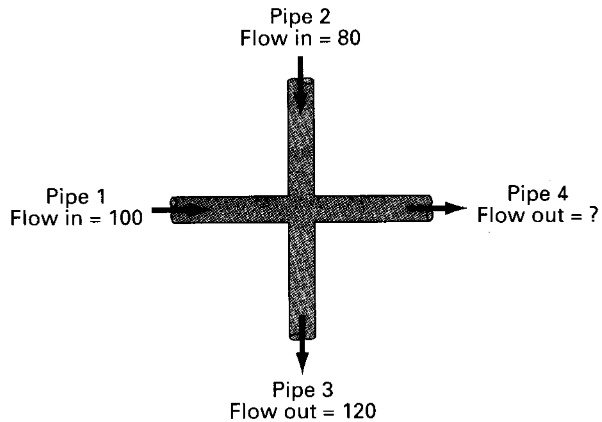


FIGURE 1.6
A flow balance for steady incompressible fluid flow at the junction of pipes.

Thus, if no change occurs, the increases and decreases must be in balance. This case, which is also given a special name—the *steady-state* computation—has many applications in engineering. For example, for steady-state incompressible fluid flow in pipes, the flow into a junction must be balanced by flow going out, as in

$$\text{Flow in} = \text{flow out}$$

For the junction in Fig. 1.6, the balance can be used to compute that the flow out of the fourth pipe must be 60.

For the falling parachutist, steady-state conditions would correspond to the case where the net force was zero, or [Eq. (1.8) with $dv/dt = 0$]

$$mg = cv \tag{1.15}$$

Thus, at steady state, the downward and upward forces are in balance and Eq. (1.15) can be solved for the terminal velocity

$$v = \frac{mg}{c}$$

Although Eqs. (1.13) and (1.14) might appear trivially simple, they embody the two fundamental ways that conservation laws are employed in engineering. As such, they will form an important part of our efforts in subsequent chapters to illustrate the connection between numerical methods and engineering. Our primary vehicles for making this connection are the engineering applications that appear at the end of each part of this book.

Table 1.1 summarizes some of the simple engineering models and associated conservation laws that will form the basis for many of these engineering applications. Most of the chemical engineering applications will focus on mass balances for reactors. The mass balance is derived from the conservation of mass. It specifies that the change of mass of a chemical in the reactor depends on the amount of mass flowing in minus the mass flowing out.

Both the civil and mechanical engineering applications will focus on models developed from the conservation of momentum. For civil engineering, force balances are utilized to analyze structures such as the simple truss in Table 1.1. The same principles are employed for the mechanical engineering applications to analyze the transient up-and-down motion or vibrations of an automobile.

TABLE 1.1 Devices and types of balances that are commonly used in the four major areas of engineering. For each case, the conservation law upon which the balance is based is specified.

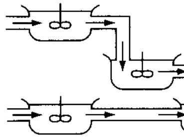
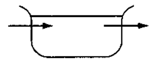
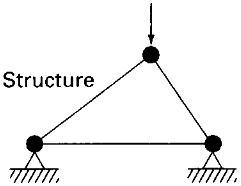
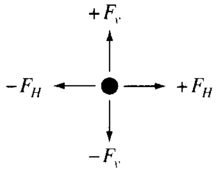
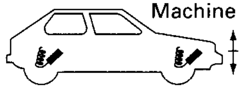
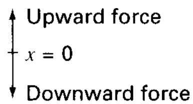
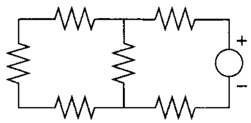
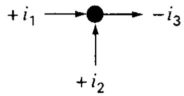
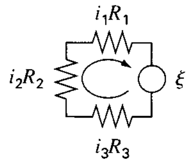
Field	Device	Organizing Principle	Mathematical Expression
Chemical engineering	 Reactors	Conservation of mass	Mass balance:  Input → Output Over a unit of time period Δmass = inputs − outputs
Civil engineering	 Structure	Conservation of momentum	Force balance:  At each node Σ horizontal forces $(F_H) = 0$ Σ vertical forces $(F_V) = 0$
Mechanical engineering	 Machine	Conservation of momentum	Force balance:  $m\dfrac{d^2x}{dt^2}$ = downward force − upward force
Electrical engineering	 Circuit	Conservation of charge	Current balance:  For each node Σ current $(i) = 0$
		Conservation of energy	Voltage balance:  Around each loop Σ emf's − Σ voltage drops for resistors = 0 $\Sigma\,\xi - \Sigma\, iR = 0$

TABLE 1.2 Some practical issues that will be explored in the engineering applications at the end of each part of this book.

1. *Nonlinear versus linear.* Much of classical engineering depends on linearization to permit analytical solutions. Although this is often appropriate, expanded insight can often be gained if nonlinear problems are examined.
2. *Large versus small systems.* Without a computer, it is often not feasible to examine systems with over three interacting components. With computers and numerical methods, more realistic multicomponent systems can be examined.
3. *Nonideal versus ideal.* Idealized laws abound in engineering. Often there are nonidealized alternatives that are more realistic but more computationally demanding. Approximate numerical approaches can facilitate the application of these nonideal relationships.
4. *Sensitivity analysis.* Because they are so involved, many manual calculations require a great deal of time and effort for successful implementation. This sometimes discourages the analyst from implementing the multiple computations that are necessary to examine how a system responds under different conditions. Such sensitivity analyses are facilitated when numerical methods allow the computer to assume the computational burden.
5. *Design.* It is often a straightforward proposition to determine the performance of a system as a function of its parameters. It is usually more difficult to solve the inverse problem—that is, determining the parameters when the required performance is specified. Numerical methods and computers often permit this task to be implemented in an efficient manner.

Finally, the electrical engineering applications employ both current and energy balances to model electric circuits. The current balance, which results from the conservation of charge, is similar in spirit to the flow balance depicted in Fig. 1.6. Just as flow must balance at the junction of pipes, electric current must balance at the junction of electric wires. The energy balance specifies that the changes of voltage around any loop of the circuit must add up to zero. The engineering applications are designed to illustrate how numerical methods are actually employed in the engineering problem-solving process. As such, they will permit us to explore practical issues (Table 1.2) that arise in real-world applications. Making these connections between mathematical techniques such as numerical methods and engineering practice is a critical step in tapping their true potential. Careful examination of the engineering applications will help you to take this step.

PROBLEMS

1.1 Answer true or false:

(a) The value of a variable that satisfies a single equation is called the root of the equation.
(b) Finite divided differences are used to represent derivatives in approximate terms.
(c) In the precomputer era, numerical methods were widely employed because they required little computational effort.
(d) Interpolation is employed for curve-fitting problems when there is significant error associated with the data points.
(e) Mathematical models should never be used for predictive purposes.
(f) The large systems of equations, nonlinearities, and complicated geometries that are common in engineering practice are easy to solve analytically.
(g) Newton's second law is a good example of the fact that most physical laws are based on the rate of change of quantities rather than on their magnitudes.
(h) A physical interpretation of integration is the area under a curve.
(i) Numerical methods are those in which a mathematical problem is reformulated so that it can be solved by arithmetic operations.

(j) Today more attention can be paid to problem formulation and interpretation because the computer and numerical methods facilitate the solution of engineering problems.

1.2 Read the following problem descriptions and identify which area of numerical methods (as outlined in Fig. PT1.2) relates to their solution.

(a) You are responsible for determining the flows in a large interconnected network of pipes to distribute natural gas to a series of communities spread out over a 20-mi^2 area.

(b) You are performing experiments to determine the voltage drop across a resistor as a function of current. You make measurements of voltage drop for a number of different values of current. Although there is some error associated with your data points, when you plot them they suggest a smooth curvilinear relationship. You are to derive an equation to characterize this relationship.

(c) You must develop a shock-absorber system for a racing car. Newton's second law can be used to derive an equation to predict the rate of change in position of the front wheel in response to external forces. You must compute the motion of the wheel as a function of time after it hits a 6-in bump at 150 mi/h.

(d) You have to determine the annual revenues required over a 20-year period for an entertainment center to be built for a client. Money can presently be borrowed at an interest rate of 10.37 percent. Although the information to perform this estimate is contained in economics tables, values are listed only for interest rates of 10 and 11 percent.

(e) You must determine the temperature distribution on the two-dimensional surface of a flat gasket as a function of the temperatures of its edges.

(f) For the falling parachutist problem, you must determine the value of the drag coefficient in order that a 200-lb parachutist not exceed 100 mi/h within 10 s of jumping. You must make this evaluation on the basis of the analytical solution [Eq. (1.10)]. The information will be used to design a jumpsuit.

(g) You are on a survey crew and must determine the area of a field that is bounded by two roads and a meandering stream.

1.3 Give one example of an engineering problem where each of the following classes of numerical methods can come in handy. If possible, draw from your experience in class and in readings or from any professional experience you have gathered to date.

(a) Roots of equations
(b) Linear algebraic equations
(c) Curve fitting: regression and interpolation
(d) Optimization
(e) Integration
(f) Ordinary differential equations
(g) Partial differential equations

1.4 What is the two-pronged approach to engineering problem solving? Into what category should the conservation laws be placed?

1.5 What is the form of the transient conservation law? What is it for steady state?

1.6 The following information is available for a bank account:

Date	Deposits	Withdrawals	Balance
5/1			512.33
	220.13	327.26	
6/1			
	216.80	378.61	
7/1			
	350.25	106.80	
8/1			
	127.31	450.61	
9/1			

Use the conservation of cash to compute the balance on 6/1, 7/1, 8/1, and 9/1. Show each step in the computation. Is this a steady-state or a transient computation?

1.7 Give examples of conservation laws in engineering and in everyday life.

1.8 Examine your engineering textbooks and find four examples where mathematical models are used to describe the behavior of physical systems. List the independent and dependent variables as well as the parameters and forcing functions.

1.9 Verify that Eq. (1.10) is a solution of Eq. (1.9).

1.10 Repeat Example 1.2. Compute the velocity to $t = 12$ s, with a step size of **(a)** 1 and **(b)** 0.5 s. Can you make any statement regarding the errors of the calculation based on the results?

1.11 Rather than the linear relationship of Eq. (1.7), the upward force on the falling parachutist is actually nonlinear and might better be represented by a power relationship such as

$$F_U = -c'v^2$$

where c' = a second-order drag coefficient (kg/m). Using this relationship, repeat the calculation in Example 1.2 with the same initial condition and parameter values. Use a value of 0.23 kg/m for c'.

1.12 The amount of a uniformly distributed radioactive contaminant contained in a closed reactor is measured by its concentration c (becquerel/liter, or Bq/L). The contaminant decreases at a decay rate proportional to its concentration; that is,

$$\text{Decay rate} = -kc$$

where k = a constant with units of day^{-1}. Therefore, according to Eq. (1.13), a mass balance for the reactor can be written as

$$\frac{dc}{dt} = -kc$$

$$\begin{pmatrix}\text{change} \\ \text{in mass}\end{pmatrix} = \begin{pmatrix}\text{decrease} \\ \text{by decay}\end{pmatrix}$$

Use numerical methods to solve this equation from $t = 0$ to 1 d, with $k = 0.1\ \text{d}^{-1}$. Employ a step size of $\Delta t = 0.1$. The concentration at $t = 0$ is 10 Bq/L.

1.13 A storage tank contains a liquid at depth y, where $y = 0$ when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are resupplied at a sinusoidal rate $3Q \sin^2 t$ (see Fig. P1.13).

Equation (1.13) can be written for this system as

$$\frac{d(Ay)}{dt} = 3Q\sin^2 t - Q$$

$$\begin{pmatrix}\text{change in} \\ \text{volume}\end{pmatrix} = (\text{inflow}) - (\text{outflow})$$

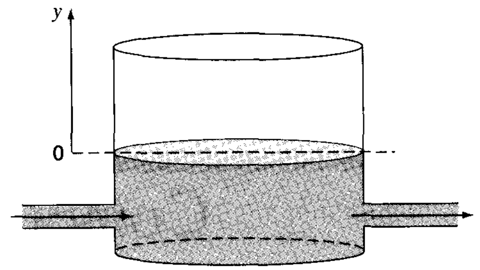


FIGURE P1.13

or, since the surface area, A, is constant

$$\frac{dy}{dt} = 3\frac{Q}{A}\sin^2 t - \frac{Q}{A}$$

Use a numerical method to solve for the height y from $t = 0$ to 5 d with a step size of 0.5 d. The parameter values are $A = 1200\ \text{m}^2$ and $Q = 400\ \text{m}^3/\text{d}$.