Mathematical Modeling, 2010

January 7, 2010

- Introduction. Great Models (Copenicus, Newton, Lagrange, Einstein)

  Simplicity of a model, non-uniqueness

  Mental experiment versus real experiment

  Drawing,algebra, ODE, PDE

- Dynamical models:

  Population growth, equation with delay, simulation, age groups.

  Parashute

- Curve fitting model

- Biological model.

1 Models and optimization

1.1 Cleat hitch

1. Consider a round surface and a rope around it. The rope is stretched at one free end by the force F_0, and is fasten at the other end, $360°$ from the other end. Assume that the tension force is proportional to the normal force at the surface (Coulomb/Amonton friction law). Compute the force in the rope along it.

The normal tension N is

$$N(s) = k(s)F(s)$$

where $k(s)$ is curvature, and s is the arch length.

The friction force FF is proportional to the normal tension

$$d\,FF(s) = -\gamma k(s)F(s)\,ds$$

and it is directed along the tangent. The tangent component satisfies the equation

$$\frac{dF}{ds} = -\gamma k(s)F(s)$$

If K is constant (round) the force decrease exponentially.

2. Design a surface that correspond to constant normal force in the rope along its path. We find $k(s)$ from the condition

$$k(s)F(s) = c = \text{const}$$

then

$$\frac{dF}{ds} = -\gamma c, \quad F(s) = F_0 - \gamma c\, s$$

and

$$k(s) = \frac{c}{F(s)} = \frac{c}{F_0 - \gamma c\, s}$$

1.2 Isochrone

2 Nonstandard equations

2.1 "Population dynamics"

Three basis models

Delay: birthrate depends on the size f population in the past.

$$\frac{d\,u}{d\,t} = u(t)(a - b\,u(t-\tau))$$

Sensus linaer model, add nonlinearity, play with

Disease spread. Equatons:

http://plus.maths.org/issue14/features/diseases/

2.2 Domino

Boundary condition. Slinky

Chain dynamics. Geisers.

2.3 Traffic wave. Continuum limit

Chain dymanics. Dis Delay

3 Instabilities

3.1 box bumping