DELAY DIFFERENTIAL EQUATIONS IN SINGLE SPECIES DYNAMICS

Source: Research paper
http://www.math.miami.edu/~ruan/MyPapers/Ruan-nato.pdf

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Matrix population model (Leslie matrix)

Jump to: navigation, search
In applied mathematics, the **Leslie matrix** is a discrete, age-structured model of population growth that is very popular in population ecology. It was invented by and named after Patrick H. Leslie. The Leslie matrix (also called the Leslie Model) is one of the best known ways to describe the growth of populations (and their projected age distribution), in which a population is closed to migration and where only one sex, usually the female, is considered.

Sources:

http://en.wikipedia.org/wiki/Leslie_matrix

Demographic research
http://www.demographic-research.org/volumes/vol12/3/12-3.pdf

Model with human dinosaurs and elephants
http://amrita.vlab.co.in/?sub=3&brch=65&sim=183&cnt=1

Linear algebra of Leslie model
http://online.redwoods.cc.ca.us/instruct/darnold/linalg/leslie2/context-leslie2-p.pdf

https://www.math.duke.edu//education/modules2/materials/linalg/leslie/lesl1.html

Theorems about Leslie Matrices:

1. A Leslie matrix **L** has a unique positive eigenvalue $\mathbf{lambda_1}$. This eigenvalue has multiplicity 1, and it has an eigenvector $\mathbf{x_1}$ whose entries are all positive.
2. If $\mathbf{lambda_1}$ is the unique positive eigenvalue of **L**, and $\mathbf{lambda_i}$ is any other eigenvalue (real or complex), then $\mathbf{|lambda_i| \le lambda_1}$. That is, $\mathbf{lambda_1}$ is a *dominant eigenvalue*.
3. If any two successive entries $\mathbf{a_j}$ and $\mathbf{a_{j+1}}$ of the first row of **L** are both positive, then $\mathbf{|lambda_i| < lambda_1}$ for every other eigenvalue. That is, if the females in two successive age classes are fertile (almost always the case in any realistic population) then $\mathbf{lambda_1}$ is a *strictly dominant eigenvalue*.
4. Let $\mathbf{x^{(k)}}$ denote the state vector $\mathbf{L^k x^{(0)}}$ after **k** growth periods. If $\mathbf{lambda_1}$ is a strictly dominant eigenvalue, then for large values of **k**, $\mathbf{x^{(k+1)}}$ is approximately $\mathbf{lambda_1 x^{(k)}}$, no matter what the starting state $\mathbf{x^{(0)}}$. That is, as **k** becomes large, successive state vectors become more and more like an eigenvector for $\mathbf{lambda_1}$.

https://www.math.duke.edu//education/modules2/materials/linalg/leslie/contents.html
A research paper
http://www.wec.ufl.edu/faculty/olim/Reprints_Oli/Oli&Zinner_2001_Oikos.pdf
A book:

Charlesworth, B. (1994) **Evolution in Age-Structured Populations** (**Cambridge** Univ. Press, Cambridge, U.K.)
Can be find in Google books:
http://books.google.com/books?id=mM_PIV_kdysC&printsec=frontcover&dq=Evolution+in+age-structured+population.+Cambridge.&hl=en&sa=X&ei=-pLzUKK3C4S8igK6lIGoCA&ved=0CC0Q6AEwAA#v=onepage&q=Evolution%20in%20age-structured%20population.%20Cambridge.&f=false