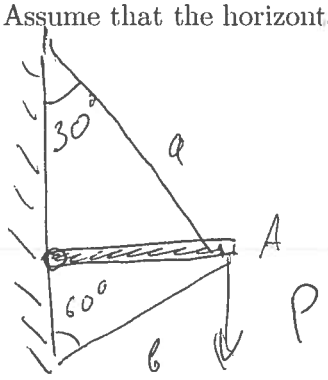
Projects: Designs and metamaterials

1 Optimal design of a frame

Consider a frame (see the picture below). Find a distribution of an elastic material between the rods a and b that minimizes the total weight, if
(a) the stiffness (the displacement in the point A of the applied force) is fixed.
or
(b) the strength of each rod is limited.

Assume that the horizontal bar is rigid.



Source:
Introduction to Structural Optimization by G.M.L.Gladwell
Google: An-Introduction-to-Structural-Optimization.pdf

2 Robust design

Consider a vertical elastic bar of crossection S_0 length L_0 that supports a hanging mass M. The bar is reinforced with (a) three or (b) four equal bars of equal crossection S, so that the frame remains stiff if an additional horizontal force F is applied to the mass; the magnitude $|F|$ of the force is equal to one, $|F| = 1$, and its direction is unknown. The deflection u of the

point A depends on F. Find the positions $Z = [z_1, z_2, ...]$ of 3 or 4 reinforcing bars that minimize the maximal (among all possible forces) deflection u in the direction of the force,

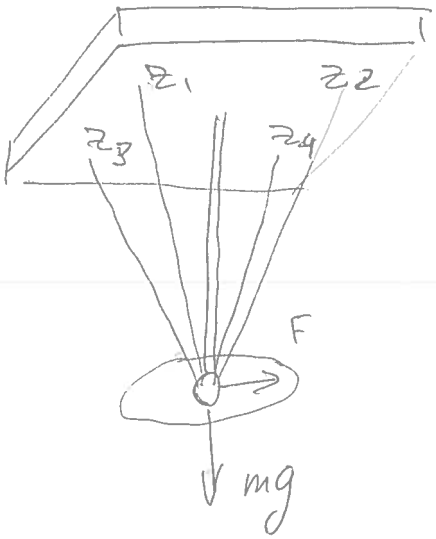
$$\min_{Z} \max_{F:|F|=1} F^T \cdot u(F, Z)$$

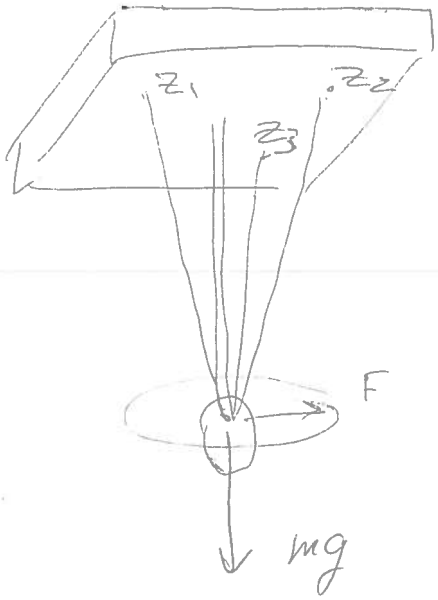
if the volume V

$$V = \sum V_i, \quad V_i = L_i S_i, \quad L_i = \sqrt{L_0^2 + |z|^2}$$

of the additional bars is fixed.

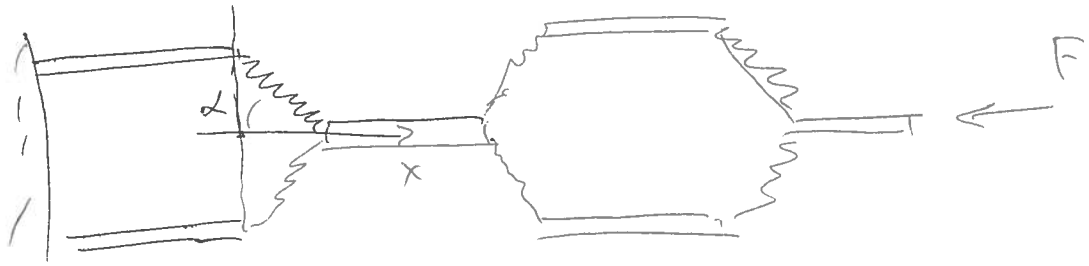
Source: http://www.math.utah.edu/ elena/publ/principal_compl.pdf





3 Chain with bi-mode transitional links

Describe contraction of the 3 links chain shown in the Figure 3. The springs are linearlyelastic, the bars are rigid and move only horizontally. Find the zones of stable elongation and zones of instability. Present the dependence of the force of the elongation and prehistory. Discuss a generalization to infinitely many links.

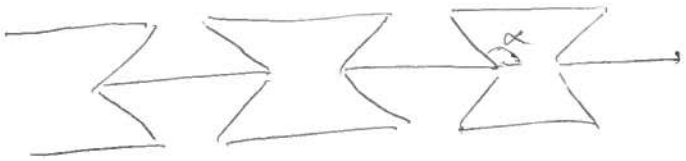


4 Metamaterials

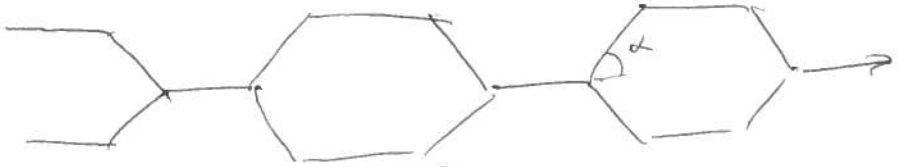
Invesigate the mode of deformation of the two structures and compute their Poisson ratio in dependence of the angle α. http://silver.neep.wisc.edu/ lakes/Poisson.html http://en.wikipedia.org/wiki/Auxetics Find other models of Auxetics sand disuse their common properties.

Suggest a 3d model of auxetic material, compte its properties. What are natural boundaries for the Poisson ratio?

A



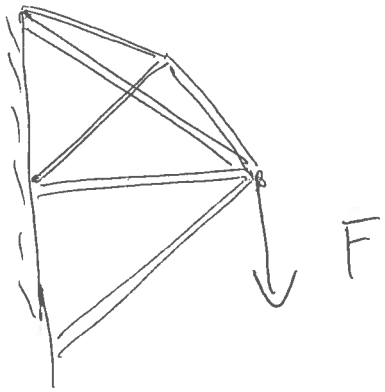
B



5 Damage evolution

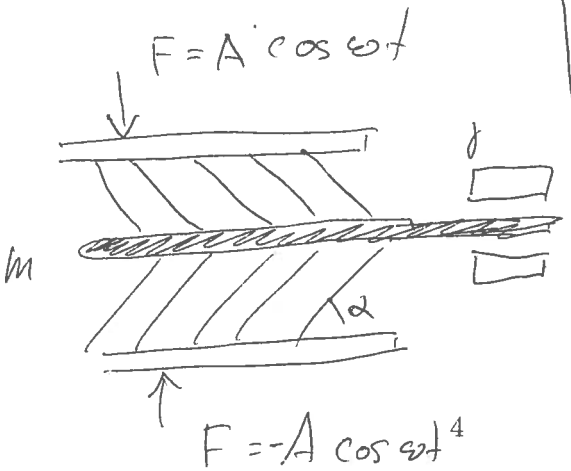
Consider a frame loaded by an increasing load P. When the load reaches a threshold value, one of the rods is overstressed, it breaks and is removed from the frame. The new frame is loaded again until the next rod breaks. The sequence of breakages depend on the design and can be controlled by changing the rods' thickness.

Build the graph of transitions between the states of the partially damaged frame. Suggest an algorithm to compute the energy needed to break the frame completely (the energy is released only in the breakage).



6 Transformer of vibrations

A device (a metamaterial) that transforms the vibration energy into a heat (or into electricity) is shown in the Figure. It consists of the extremal massless and rigid shell (two rods), the inclined massless rigid joins that transform vertical vibrations into a horizontal motion $x(t)$ of the central rigid rod with the mass m, and the dissipation device (or a generator) which resistance force $F_D = -\gamma\dot{x}$



Derive the eqs of the motion, Find the best values of m, ω to maximize the dissipation.