Population dynamics 1

A group of models describes growth of population size of one isolated species.

1. Malthusian growth accounts for size increase due to difference between birth and death rates,

2. Logistic models http://en.wikipedia.org/wiki/Logistic_function accounts for limiting caring capacity of the habitat.The same model describes spread of innovations, new products, new words in language, rumors, etc. (find the references)

3. Allee model http://en.wikipedia.org/wiki/Allee_effect
accounts also for the positive correlation between population size or density and the mean individual fitness (oftentimes measured as *per capita* population growth rate) of a population or species.

The models can be modified to account for delay (see for example http://www.math.miami.edu/~ruan/MyPapers/Ruan-nato.pdf).

The diffusive logistic growth (DLG) model describes spatial spread of a species. (find the references)

These models are simulated by either differential or difference schemes. The difference scheme may be unstable if the step size is too large.

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Population dynamics: simulation 1

Here the malthusian model, logistic growth, logistic growth with delay, Allee models are simulated as differential and difference equations.

Differential equations:
First plot shows numerical solution to Logistic equation.

```
> with(plots);
> p := dsolve({y(0) = 1, (D(y))(x) = y(x)*(3-y(x))}, type = numeric, range = 0 .. 2);
> odeplot(p);
```

Analytic solution is given as:

```
sl2 := dsolve({diff(y(t), t) = a*(1-y(t)/T)*y(t), y(0) = P});
```

Allee equation.

The graphs show different behavior (growth or decay), depending of whether the initial condition is above or below the threshold.

```
p1:= dsolve({D(y)(x) = (y(x)-1)*y(x)*(3-y(x)), y(0)=1.05}, type=numeric, range=0..3):
p2:= dsolve({D(y)(x) = (y(x)-1)*y(x)*(3-y(x)), y(0)=.95}, type=numeric, range=0..3):
r1:=odeplot(p1); r2:=odeplot(p2);
display({r1, r2});
```

Next graph shows the growth rate in the Allee equation, dependence of the population size.

```
v:=(y-1)*y*(3-y);
plot(v, y=0 .. 3.2);
```

The same phenomena are modeled by Difference equations:

Malthusian model

```
> p[0] := 1;                    1
> for i from 0 to 25 do p[i+1] := 1.07*p[i] end do;
> pointplot({seq([i, p[i]], i = 1 .. 20)});
```

Logistic model

```
> q[0] := .1; for i from 0 to 40 do q[i+1] := q[i]+(.3*(1-(1/20)*q[i]))*q[i] end do;
> pointplot({seq([i, q[i]], i = 1 .. 40)});
```

Logistic model with delay (the difference schemes are more flexible)

```
> qd[0] := 1; qd[-1] := 1; qd[-2] := 1; for i from 0 to 30 do qd[i+1] := qd[i]+(.3*(1-
(qd[i-1]+qd[i-2])*(1/10)))*qd[i] end do;
> pointplot({seq([i, qd[i]], i = 2 .. 30)});
```

Allee model

```
> pla[0] := 1.4; for i from 0 to 40 do pla[i+1] := pla[i]-(.3*(1-pla[i]/(1.2)))*(1-(1/6)*pla[i])*pla[i] end do;
> pointplot({seq([i, pla[i]], i = 1 .. 40)});
```

Depending on the growth rate, the discrete model may become unstable due to numerical instabilities. Vary the coefficients and observe this instability.