HW Evolutionary Games

Due at March 7, 2014

Problem 1 Consider evolution of three species:

G (Good neighbors) The population of the size P_G has original average fitness $F_1 = 1$, and the fitness increases k times ($k = 1.2$) in each time period. In each time period, a certain portion p of agents G becomes sick and looses a part of its fitness, which becomes $F_2 = .333$, ($F_2 < F_1$). The agents in G group are helping the sick agents, restoring their fitness back to $F_1 = 1$. In the process, they loose a part of their own fitness that becomes $F_3 = .667$, where $F1 > F3 > F2$. If sick agents are not helped, they die in the next time period, their fitness becomes zero, $F_4 = 0$.

C (Cheaters) Agents in C behave exactly like agents in G, except C never help anyone. The G agents are helping the sick C agents in the same manner as they are helping sick G agents, restoring their fitness to $F_1$and loosing a part of their fitness that becomes F_3.

J (Judges) Agents J behave as G and C, with one difference: The help only those who helps, namely only G and J and they do not help C.

Build an evolutionary model and replicator equation that fits the description, set the parameters, and model discrete dynamics for the following initial conditions

a) $P_G = 10,\ P_C = 1,\ P_J = 0$

b) $P_G = 10,\ P_C = 3,\ P_J = 2$

Problem 2. **Stability of evolution** Consider the rock-scissor-paper game with a slightly alternated payoff matrix

$$A = \begin{pmatrix} 1+\epsilon & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$$

where ϵ is a small number, $|\epsilon| \ll 1$, in continuous time.

Build the replicator equation and model the dynamics starting from the point $1/2, 1/4, 1/4$. Does the trajectory converge to a stable point? Consider two cases, $\epsilon > 0$ and $\epsilon >< 0$.