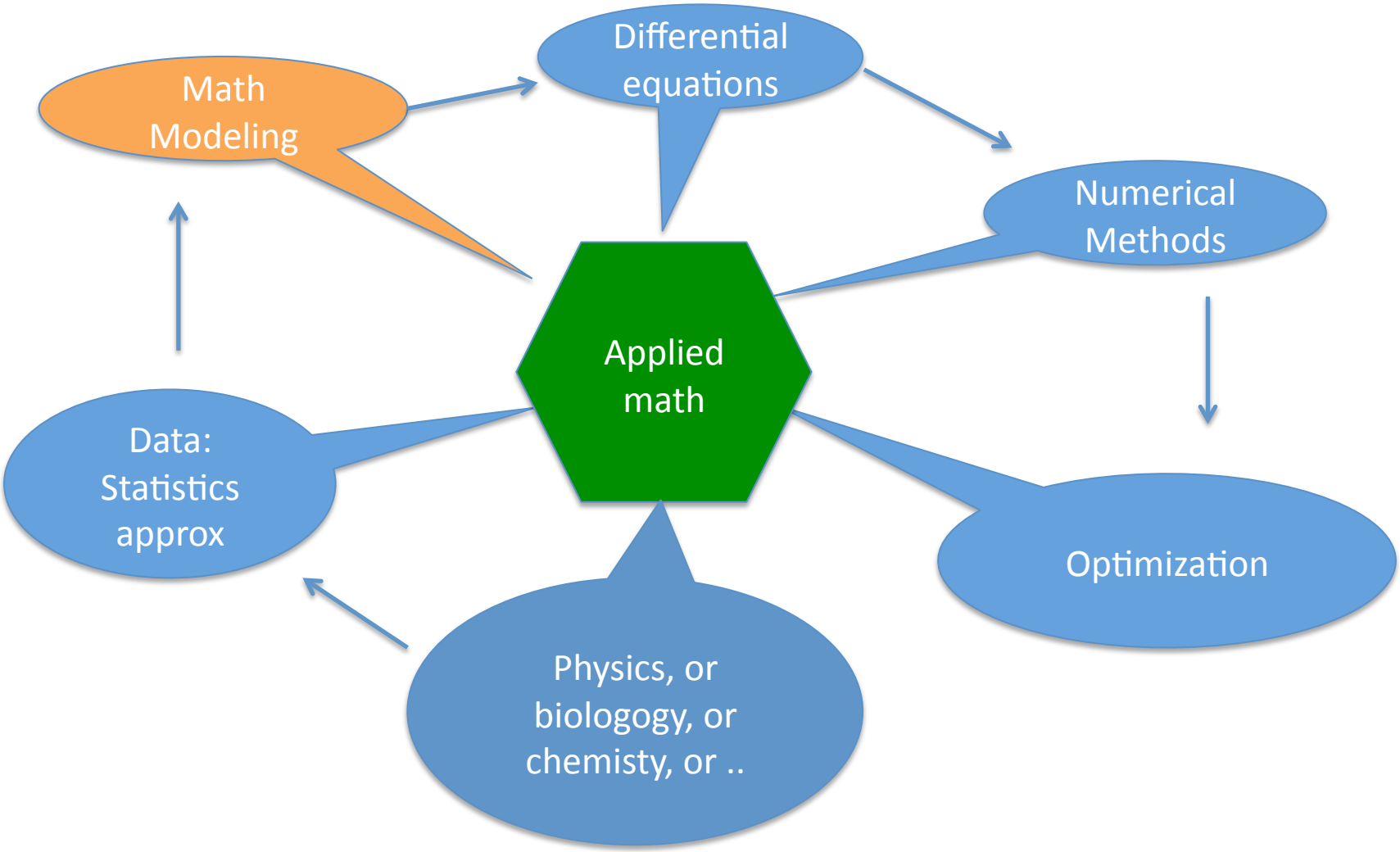
MATH 5740/MATH 6870 001
MATH MODELING 2013
09:40 AM-10:30 AM JWB 208
Andrej Cherkaev

Introduction

Components of applied math



Models and reality

- *Theory attracts practice as the magnet attracts iron.*
  Gauss

- The applied math studies problems that are inspired by a nature phenomenon or a social need.
- We live in the world of models:
  - Great models: Universe, Nuclear Physics, Evolution, Social organization – determine our life forcing our judgment, decisions, and feelings
  - Paradigms arrive in the form of new models: atoms, Neo-Darwinism, fractals, democracy.
- Models as reference points: New ideas are placed in the framework of exiting models

Definition from Wiki

- A **mathematical model** uses mathematical language to describe a system.
- Mathematical models are used in the natural sciences and engineering disciplines (such as physics, biology, earth science, meteorology, and engineering) and in the social sciences (such as economics, psychology, sociology and political science)..
- The process of developing a mathematical model is termed 'mathematical modelling' (also modeling).

Galilei, Galileo (1564 - 1642):

[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.

Opere Il Saggiatore p. 171.

Thought_experiment
http://en.wikipedia.org/wiki/Thought_experiment

Features of math models

Goals: descriptive or design/optimization (natural or engineering model).

Range of applicability (Ideal gas, Black matter, optimization)

Validation: Mental experiment vs. real experiment. Galileo.

Results: Numerical or analytic. Simulation models.

Predictability: Curve fitting vs. equation solving.

(based on a priori principles or empirical). The "good" model should to a degree predict the behavior of a system in a new environment.

Mathematical tools

Geometry, Equation solving, ODE, PDE, calculis of variations, numerical methods, game theory, probability, statistics, optimization and control theory, etc.

Creativity

Mathematics is made of 50% formulas, 50% proofs, and 50% imagination.

Model is an intentionally distorted system description that emphasizes desirable features

Everything should be made as simple as possible, but not simpler. ***A. Einstein***

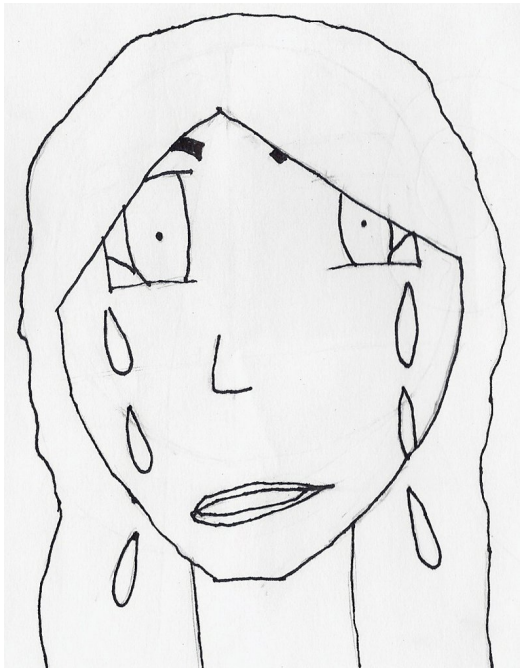
$\Rightarrow$ Model is not unique

- Bridge: for traffic model – a piece of road
- For flight navigation model – an obstacle
- For vibration model – an unbreakable elastic structure
- For strength model – an elastic-plastic structure
- For bungle jumping -- a base.
- For an artist – a shape.

The Millau Viaduct

Models intentionally distort reality

No one needs a geographical map in the scale 1:1



Syllabus (Wish list)

Introduction. Principles of modeling. Great Model of the Universe. From observations and assumptions to equations.

How to split the class in several working groups

Growth and interaction of species. Population dynamics. Epidemic spread. Model of population dynamics. Model of epidemic disease and vaccination. PDE Model of population dynamics.

Wave models. Traffic: shock waves, stabilization factors. Simulation of a traffic jam. Domino, slinky, and similar “discrete waves”

Modeling of conflicts: Games- Best strategy, minimax. Cooperative game - How to share fairly. Evolutionary games - Models of social behavior.

Models with graphs

Models with shapes.

Lattice models of complicated structures. Metamaterials

Principles of modeling in the syllabus

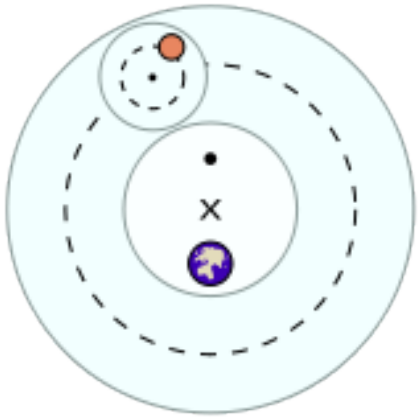
- Simplification. To what extend?
- Combination of discrete and continuum descriptions. Examples of continuum models: Population dynamics, Traffic waves. Examples of discrete description: Domino train, breakable chains, division of the class.
- Waves and repeated sequences of events. -Wave of damage, traffic waves, evolution dynamics.
- Stability and catastrophes -- Domino, multiple equilibria system, breakable chains.
- Multiscale approach, models for different scales: Evolution dynamics, decease spread.
- Descriptive modeling vs Design and Optimization.
- Formalization of ``humanitarian'' concepts.

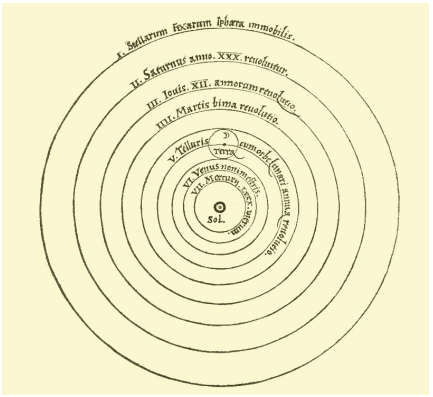
Organization of the class work

- For the project work the class will be divided into groups of tree
- A presenter will orally present the group's work, the report will be written. The group get a grade for the project.
- The groups will be formed for each project, the roles will be reassigned so that each student will be researcher and presenter.

- **Problem: (one week)**
- Write an algorithm for dividing the class into groups of three for each project. During the semester, each student should work with all class mates.
- Next time, bring in your ideas.

Models of the Universe

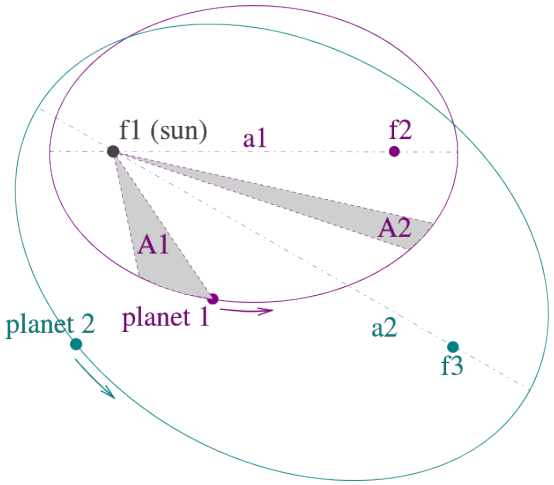




a. Turtles all the way down.
b. Ptolemy: Earth in the center, planets move with constant speed, circular orbits, and epicycles: http://www.pbs.org/wgbh/nova/ancient/ancient-computer.html
c. Copernicus 1473-1543: Sun in the center, planets move with constant speed, circular orbits, and epicycles.

http://en.wikipedia.org/wiki/Turtles_all_the_way_down
http://en.wikipedia.org/wiki/Geocentric_model
http://en.wikipedia.org/wiki/Copernican_heliocentrism

Kepler's model, Newton' law



Johannes Kepler published his
first two laws in 1609, the third –in 1619

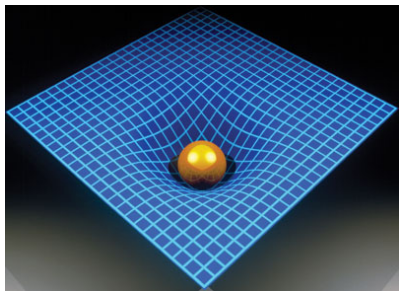
(1) The orbits are ellipses, with focal points $f1$ and $f2$ for the first planet and $f1$ and $f3$ for the second planet. The Sun is placed in focal point $f1$.

(2) (2) The two shaded sectors A1 and A2 have the same surface area and the time for planet 1 to cover segment A1 is equal to the time to cover segment A2.

(3) (3) The total orbit times for planet 1 and planet 2 have a ratio a1^3/2 : a2^3/2.

$$\mathbf{F}_{12} = \gamma \frac{m_1 m_2}{|\mathbf{r}_{12}|^3} \mathbf{r}_{12}$$

Newton gravity law explained all three Kepler's laws
but left open the question of how the gravity force acts.



Einstein's general relativity explains the nature of the gravity force

History of modeling of the Universe (HW 1)

Write a short essay (< 3 pages) about models of Universe:

- *Turtles all the way down*
- *Ptolemy* (Klaúdios Ptolemaîos),
- *Nicolaus Copernicus*
- *Tycho Brahe* and *Johannes Kepler,*
- *Isaac Newton* (gravity law)
- *Albert Einstein* (general relativity)
- *Big Bang theory*

Characterize the models as

empirical,
data fitting,
equation solving,
general-principle-based

and comment on motivation for the improvement/development. thought and physical experiments. What question were asked?

Use Internet, Wiki, or any other available sources

http://astro.unl.edu/naap/ssm/animations/ptolemaic.swf