How work assignments for M- 5720

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1 HW 1. Symmetrization, Euler equation

1. Derive the equations of a symmetrizated ellipse.

2. Derive the Euler equation and solve the problem of the best approximation of a discontinuous function

$$\phi(x) = \begin{cases} 0, & \text{if } 0 \le x < 1 \\ 1, & \text{if } 1 \le x < 2 \end{cases}$$

   by a smooth function $u = u(x)$.

   The variational problem to consider is

$$\min_{u(x)} \int_0^2 \left(\alpha^2 (u')^2 + (u - \phi)^2\right) dx$$

   where α^2 is a "penalty for variation" of the estimating function u. Solve the Euler equation and graph (using Maple) the familt of solutions for the values of α: 0.01, 0.1, 1, 10, 100.

2 HW 2. Euler equation (cont), natural boundary conditions, Weierstrass test

1. Derive the Euler equation and Natural boundary conditions to the variational problem

$$\min_{u(x)} \int_a^b F(x, u, u', u'')\, dx$$

   adapting the procedure for the deriving of the Euler equation to Lagrangian $F(x, u, u', u'')$.

2. Check that Euler equation correspond to minimum (not maximum of saddle point) of the problem

$$\min_{u(x)} \int_a^b F \frac{\sqrt{1+(u')^2}}{v(x,u)} dx, \quad v(x,u) \geq 0,$$

use the Weierstrass test.