Midterm exam Math 5710 Fall 2013

Your name.....

October 23, 2013

1. The electrical circuit is given in Figure 1. Find the current between nodes A and B. Show algebraic formulas.

2. Show, computing *curl* v, that
(a) $v_1 = (y^2, x)$ is not the gradient of any function, but

$$(b) \qquad v_2 = \left(\frac{2x}{y}, -\frac{x^2}{y^2} \right)$$

is. Find the potential.

3. Can vector field $v_3 = (xy, x)$ be represented as $v_3 = curl\, w$ where w is some vector field?

4. The force field F depends on the coordinates (x, y, z) as $F = (y^2 z, 2x\, y\, z, ?)$. What is the third component of the force, if it is known that F is conservative $F = \nabla u$.

5. In pure rotation, the velocity field v is represented as $v = (\omega_1, \omega_2, \omega_3) \times (x, y, z)$. Compute $curl\, v$, $div\, v$. Show that v and ω are perpendicular.

6. Compute $\nabla \cdot (u \times v)$, where u, v are two vectors.