List of course projects (Preliminary)

Take one (First come – first served)

September 24, 2003

These are drafts of the course projects. Please feel free to modify them replacing electrical network with mechanical one, adding new goals, etc. A group of two can take one project or modify two close projects; the grade will be shared.

1 Project. Vibrating chains

Sequential connections Consider a chain of $N = 10$ large masses M_+ alternating with the small masses $M_- = \frac{1}{2}M_+$. The masses are separated by the linear spring with stiffness C. A periodic force $F(t) = \cos(\omega t)$ is applied to the left end of the chain, the right end is attached to the mass M_0.

Find the motion of the right end of the chain. Graph the dependence of the frequency of oscillations ω. Choose the frequency ω to minimize and to maximize the norm $N(u_0)$ of deflection u_0 of the mass M_0,

$$N(u_0) = \frac{1}{2\pi\omega} \int_0^{2\pi\omega} u_0^2 dt$$

Parallel connections Consider a chain of $N = 10$ sequential large masses M_+ with the linear spring with stiffness C between them. To each mass M_+ in parallel is attached a spring C_- with the small masses M_- at the end. A periodic force $F(t) = \cos(\omega t)$ is applied to the left end of the chain, the right end is attached to the mass M_0.

Find the motion of the right end of the chain. Graph the dependence of the frequency of oscillations ω. Choose the frequency ω to minimize and to maximize the norm $N(u_0)$.

2 Network design

Consider an electrical network of 5×5 nodes joined vertically and horizontally by resistances R_{nm} such that

$$R_- \leq R_{nm} \leq R_+.$$

The left lower node is grounded (has zero potential).

Excitation Two type of excitations are considered:

(1) One can apply the voltage sources on any boundary node or any combination of them.

or

(2) One can apply the current sources on any boundary node or any combination of them provided that the sum of currents is zero.

2.1 Project. Sources optimization: Minimization of the whole resistivity

Assume that the resistances R_{nm} are equal. The resistance of the whole network is defined as the energy dissipated in it. The sum of absolute values of the applied currents is equal to one.

1. How to distribute the current between the nodes to*minimize* the resistance of the whole network.

2. How to distribute the current between the nodes to *maximize* the resistance of the whole network.

2.2 Project. Sources optimization: Maximization of the current in a link

Assume that the resistances R_{nm} are equal. Any potential source can be applied to any boundary node but the grounded one, the sum of squares of these potentials is equal to one.

What potentials should be applied to maximize the current through the vertical link in the center of the network.

Assume in addition that the potentials must be applied only on the lower boundary of the network. What potentials should be applied to maximize the current through the vertical and horizontal links in the center of the upper side of the network.

2.3 Project. Detection of the broken link

Assume that that the resistances R_{nm} are equal but one link is broken. Apply any boundary sources as many times as you wish to detect the position of the broken link.

How to minimize the number of measurements?

2.4 Project. Network design

The network transports the currents from the left to the right side. The total current to be transported through the network equals one. The equal positive current is applied to all nodes at the left side and the equal negative current – to all nodes at the right side.

Choose the resistivity of the links in a network (subject to constraints $R_- \leq R_{nm} \leq R_+$) that

1. Maximizes the current through a vertical link at the center of the network.

2. Maximizes the potential difference between two inner nodes

2.5 Project. Network design

The network transports the currents from the left to the right side. The total current to be transported through the network equals one. The equal positive current is applied to all nodes at the left side and the equal negative current – to all nodes at the right side.

Assume that N links have the resistivity R_- and the rest have the resistivity R_+.

1. Maximize the resistance of the whole network.

2. Minimize the resistance of the whole network.

2.6 Complex-valued conductivity

Replace the resistors around the main diagonal with the capacitors of equal capacity C. Assume that the harmonic current source $I(t) = I_0 \sin(\omega t)$ is applied to a node in the right side and current $-I(t)$ is applied to one node at the left side.

Derive the real-valued system for the network and compute the currents in the network. Derive the quadratic function which minimization leads to the solution of the network.