

Math 5710  
Applied mathematics

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## Rules

**Class meets** at 10:45 MWF

**Office hours:** After the class and by appointment

The class is **Addressed** to graduate and senior undergraduate students in math, science, and engineering. Grade is based on regular homework assignments and a course project.

**Text:** Gilbert Strang. Introduction to Applied Mathematics. Instructors notes (will be distributed).

**Syllabus:**

- 1.G. Strang: Chapters 1,2, 3
2. Notes: Extremal problems.

The **grade** will be based on homework scores, in-class exam, and course project. As a rule, the homework will be assigned each week.

## What is Applied mathematics?

Applied math is a group of methods aimed to solve problems in sciences, engineering, economics. Examples: Mathematical physics, Mathematical biology, Control theory, Aerospace engineering, planning, math finance.

Generally, applied math deals with complicated objects, allow many ways to approach and describe the problem, manipulation of objectives, variety of methods. Success is a new elegant simple model that catch the phenomenon and novel clever methods to solve it.

There is a fuzzy boundary between applied math and engineering and, at the other side, applied and pure mathematics. Applied math discovers new problems which could become subjects of pure math (like geodesics), or develop to a new engineering discipline (like elasticity theory).

Requires: Expertise in many areas of mathematics and science, engineering intuition and common sense, and collaboration skills.

## Methods of applied math

1. Data proceeding and formulation of the problem: Math Modelling and Statistics. There is a limited freedom of choice of the model.

The art of simplification to make the problem solvable but not trivial. Often we deal with large system (thousands of variables, equations and inequalities). No one need a one-to-one map. A clever model separates the main phenomenon and allows for analytic treatment, followed by extensive numerical development.

Models have ranges of applicability (Ideal gas and black holes)

Numerical models allow to solve classical problems and address novel classes of problems (like math finance, math genetics, weather prediction). Development of the theory of free boundary problems was caused by the possibility of numerical solution the problem in an arbitrary domain.

2. Method of solutions (What causes the effect?): Linear algebra, differential and integral equations, approximation theory, variational principles, numerical methods. These will be intensively discussed in the course. The same physical problem can be approached differently, using either statistics, of differential equations, or variational methods, or a combination of them. Examples: Weather forecast; conflict situation: Worst case scenario or average outcome.
3. Improvement and recommendation (How to maximize an objective?): Optimization, variational methods, control theory, game theory. Optimization is the ultimate objective of study of an engineering problem. Sometimes the improvement is achieved by varying the parameters, but generally it is a serious math problem that will be discussed in the class.

## Example: Optimal design

**Intuitive design** In practice, the process of design always includes a mysterious element: The designer chooses the shape and materials for the construction using intuition and experience. Since ancient times this technique has proved effective, and for centuries engineering landmarks such as aqueducts, cathedrals, and ships were all built without mathematical or mechanical theories.

**Math models** However, from the time of Galileo and Hooke, engineers and mathematicians have developed theories to determine stresses, deflections, currents and temperature inside structures. This information helps in the selection of a rational choice of structural elements.

**Common-sense improvement, educated guess** Certain principles of optimality are rooted in common sense. For example, one wants to equalize the stresses in a designed elastic construction by a proper choice of the layout of materials. The overstressed parts need more reinforcement, and the understressed parts can be lightened. These simple principles form a basis for rational construction of amazingly complicated mechanical structures, like bridges, skyscrapers, and cars. Still, knowledge of the stresses in a body is mostly used as a checking tool, parallel with the design proper, which remains the responsibility of the design engineer.

**Preconditions** In the past few decades, it has become possible to turn the design process into algorithms thanks to advances in computer technology. Large contemporary projects require the use of computer-aided design systems. These systems often incorporate algorithms that gradually improve the initial design by a suitable variation of design variables, namely, the materials' cost and layout. Optimization techniques are used to effect changes in a design to make it stronger, lighter, or more reliable.

**Theory is developing** This progress has stimulated an interest in the mathematical foundations of structural optimization. These foundations are the main topic of this book. The theory of extremal problems is used to address problems of design. A design problem asks for the best geometry of layouts of different materials in a given domain. Of course, this approach simplifies (or, as a mathematician would say, idealizes) the real engineering problem, because questions such as convenience or cost of manufacturing are not considered.

Analysis of optimal structures allows us to formulate general principles of an optimally designed construction. In particular, we can extend the intuitive principle of equally stressed construction to a multidimensional situation and find optimal structures that are, in a sense, hybrids of simple mechanisms.

The construction that adapts to the varying load has some common features of living tissues: Bridge to math biology.