Midterm 2

M 3160: Applied Complex Variable

Spring 2018

1. Evaluate the integral

$$I_1 = \int_C \frac{\exp(z)}{(z+8)(z-1)} dz, \quad C := \{z : |z| = 2\}.$$

2. Evaluate the integral

$$I_2 = \int_C \frac{z^3 + \sin(3\,z)}{(z+i)^3} dz, \quad C := \{z : |z| = 2\}$$

3. Expand function

$$F(z) = \frac{1}{1+2z}$$

into the Taylor series around the point $z_0 = 1$. What is the radius of convergence?

4. Expand function

$$f(z) = \cos(3z) - \exp\left(\frac{1}{z}\right)$$

into the Laurent series around the point $z_0 = 0$.
Find the coefficients of the expansion.
What is the radius of convergence?

5. Using representation

$$F(z) = \frac{3z-1}{(z-3)(z+1)} = \frac{2}{z-3} + \frac{1}{z+1},$$

find the Laurent series of function $F(z)$ around the point $z_0 = 0$. Define three regions where three different expansions are valid, find these expansions.

6. Compute the residue

$$\mathrm{Res}_{z=0} f(z)$$

if the function $f(z)$ is either

$$f(z) = f_a(z) = \frac{e^z + z^2}{z^3} \ (a); \quad \text{or } f(z) = f_b(z) = \exp\left(\frac{1}{z}\right) \ (b).$$

7. Indicate the type of singularity of the following functions at $z = z_0$. Is is

   (a) a removable singular point,

   (b) a pole of the order n (find n), or

   (c) An essential singular point?

   Consider the functions

   (1) $f_1(z) = \frac{1-\cos(z)}{z^2},\ z_0 = 0;$ (2) $f_2(z) = z^2 \sinh\left(\frac{1}{z^2}\right),\ z_0 = 0;$

   (3) $f_3(z) = \frac{x-\sin(z)}{z^4},\ z_0 = 0;$ (4) $f_4(z) = \dfrac{\exp(iz)}{(z-2)^2},\ z_0 = 3.$