# Midterm 2

## M 3160: Applied Complex Variable

## Spring 2018

1. Evaluate the integral

$$I_1 = \int_C \frac{\sin(z)}{(z+3)(z-1)} dz, \quad C := \{z : |z| = 2\}.$$

2. Evaluate the integral

$$I_2 = \int_C \frac{z^2 - \exp(2z)}{(z+i)^3} dz, \quad C := \{z : |z| = 2\}$$

3. Expand function
$$F(z) = \frac{1}{2 + z}$$
into the Taylor series around the point $z_0 = 1$. What is the radius of convergence?

4. Expand function
$$f(z) = \cos(2z) - \cos\left(\frac{2}{z}\right)$$
into the Laurent series around the point $z_0 = 0$.
Find the coefficients of the expansion.
What is the radius of convergence?

5. Using representation

$$F(z) = \frac{3z}{(z-1)(z+2)} = \frac{1}{z-1} + \frac{2}{z+2},$$

find the Laurent series of function $F(z)$ around the point $z_0 = 0$. Define three regions where three different expansions are valid, find these expansions.

6. Compute the residue
$$\operatorname{Res}_{z=0} f(z)$$
if the function $f(z)$ is either
$$f(z) = f_a(z) = \frac{e^{iz} - z^3}{z^4} \quad (a); \quad \text{or } f(z) = f_b(z) = \cos\left(\frac{1}{z}\right) \quad (b).$$

7. Indicate the type of singularity of the following functions at $z = z_0$. Is is

    (a) a removable singular point,

    (b) a pole of the order $n$ (find $n$), or

    (c) An essential singular point?

    Consider the functions

    (1) $f_1(z) = \frac{z - \sin(z)}{z^3}$, $z_0 = 0$;    (2) $f_2(z) = z^5 \sinh\left(\frac{1}{z^2}\right)$, $z_0 = 0$;

    (3) $f_3(z) = \frac{1 - \cos(z)}{z^3}$, $z_0 = 0$;    (4) $f_4(z) = \frac{\exp(iz)}{(z - 2)^2}$, $z_0 = 3$.