M 3160 Applied Complex Variable: Spring 2018
Final exam

Your name ....

Solve any six problems.

1. Find all roots of the equation, plot then on the complex plane.

$$z^3 + 8i = 0$$

2. The real part of a complex function $f(x, y) = u(x, y) + iv(x, y)$ is

$$u(x, y) = x^3 - 3xy^2$$

a) Check that $u(x, y)$ is a harmonic function.

b) If $f(x, y)$ is an analytic function, what is its imaginary part $v(x, y)$?

3. Solve the equation; find all solutions.

$$e^z + 1 = 0$$

4. Using Cauchy integral formula, evaluate the integrals

$$I_1 = \int_C \frac{\exp(z^2 - 10)}{(z+8)(z-1)} dz, \quad C := \{z : |z| = 3\}.$$

$$I_2 = \int_C \frac{\exp(z^2 - 10)}{(z+8)(z-4)} dz, \quad C := \{z : |z| = 3\}.$$

5. Using representation

$$F(z) = \frac{5}{(z-2)(z+3)} = \frac{1}{z-2} - \frac{1}{z+3}$$

find the Laurent series of function $F(z)$ around the point $z_0 = 0$. Define three regions where three different expansions are valid, find these expansions.

6. Using residue theorem, evaluate integral

$$I_2 = \int_0^\infty \frac{dz}{z^4 + 16} dz,$$

7. Find a linear fractional transformation that maps the horizontal line $[-\infty < x < \infty,\ y = 1]$ onto the circle $|u + iv| = 1$.