Math 3160. Complex Variables, Spring 2018.
Midterm exam 1. Version 2

Your name .....

1. Expess the absolute value and argument of w through x, y where $z = x + iy$,

$$w = \frac{1 - \bar{z}}{1 + z}$$

2. Find all roots of the equation

$$z^4 = -16$$

3. Plot the region in the $(x,\ y)$ plane, where $x = \Re\, z$ and $y = \Im\, z$ are the real and imaginary parts of z:

$$\Re\, z \geq |\Im\, z|, \quad |z| < 2$$

4. Show that f(z) is analytic, and $\Im(f(z))$ is harmonic, compute the derivative $f'(z)$.

$$f(z) = \cos(z) = \cos(x)\cosh(y) - i\sin(x)\sinh(y).$$

5. Compute

$$\log z, \quad \mathrm{Log}(z), \quad \text{where } z = (1-i)^4$$

6. Compute

$$(1-i)^i$$

7. Solve (find all solutions)

$$(a)\ \sin(z) = 5, \quad (b)\ \exp(z) = -1$$

8. Express $\tanh(iz)$ through $\sin(z)$ and $\cos(z)$