Make-Up Midterm. M 3150-001, Spring 2017

Your name (please print) ....

Solve the following problems. Show the method (the derivation), do not simply copy the final formulas.

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1. Find $u(x,t)$, that satisfies the following conditions:

Heat Equation	$\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2}, \quad x \in [0,2],\ t \in [0,\infty).$
Boundary cond.	$u(0,t) = 0, \quad u(2,t) = 0.$
Initial cond.	$u(x,0) = 3\sin(\pi x)\,.$

2. Determine the equilibrium temperature distribution $u(r, \theta)$ in a disk of radius a = 1, if the boundary temperature is $u(1, \theta) = 3\cos(3\theta)$. Note: Laplacian in polar coordinates has the form

$$\nabla u(r, \theta) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}.$$

3. Find the sine Fourier series of

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \\ 0 & \text{if } x \in [1,3] \end{cases}, \quad x \in (0,3).$$

Find the first three nonzero coefficients of the series.

4. A string vibrates according to the equation

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2},$$

where $u(x,t)$ is the deflection, $x \in [0,1]$, $t \in [0,\infty)$.
ends are fixed: $u(0,t) = 0$, $u(1,t) = 0$,
the initial shape $u(x,t)$ is $u(x,0) = 2\sin(5\pi x)$, and
the initial speed is zero, $\frac{\partial u}{\partial t}|_{t=0} = 0$.
Find $u(x,t)$.