Midterm 2. M 3150-001, Spring 2015

Your name (please print) ....

1. Are the following functions piece-wise differentiable? (if they are not, state the reason)

   (a)
   $$f_1(x) = |x|, \quad x \in [-1, 1];$$

   (b)
   $$f_2(x) = x \log x \quad x \in [0, 1];$$

   (c)
   $$f_3(x) = \begin{cases} x + 1 & x \in [0, 1] \\ -x & x \in [1, 2]. \end{cases}$$

   (d)
   $$f_4(x) = x^{2/3}, \quad x \in [0, 2];$$

   (e)
   $$f_5(x) = \frac{1 + x}{1 - x}, \quad x \in [0, 2].$$

2. Consider function $g(x) = e^x$, $x \in [0, 1]$. Its Fourier series expansion represents a periodic function $\hat{g}(x)$, $x \in (-\infty, \infty)$.

Find the series and sketch the graphs of $\hat{g}(x)$ for:

(a) General (sin and cosine) Fourier series expansion,

(b) Sine Fourier series expansion,

(c) Cosine Fourier series expansion.

Do not evaluate the integrals.

3. (a) Find the sine and cosine Fourier series of

$$f(x) = \begin{cases} 1 & x \in [0, 1] \\ 0 & x \in [1, 2] \end{cases}.$$

   (b) Find first two nonzero coefficients of both series.

4. The Fourier series of a function $f(x)$ is

$$\hat{f}(x) = 2 + \sum_{i=1}^{\infty} \frac{4\pi}{n^3(1 + 2n^2)} \cos(\pi n x).$$

   Find the Fourier series of $f''(x)$.

5. A string vibrates according to the equation $u_{tt} = 4u_{xx}$, where $u(x,t)$ is the deflection, $x \in [0,1]$, $t \in [0,\infty)$.

The ends are fixed: $u(0,t) = 0$, $u(1,t) = 0$,

initial shape $u(x,t)$ is $u(x,0) = 3\sin(2\pi x) - 2\sin(5\pi x)$,

initial speed is zero, $\frac{\partial u}{\partial t}\big|_{t=0} = 0$.

Find $u(x,t)$.