Introductory Quiz for m 3510 Partial Differential Eqiations

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Solve the problems below. If you cannot solve any of them, look into ODE and calculus books and refresh your skills.

The material in the quiz is essential for the course and I assume that you know it.

1. Simplify

$$2\sin^2 x + \cos(2\,x); \quad \sin(2\,x) - 2\sin(x)\cos(x) \tag{1}$$

2. $f(x)$ is a twice differentiable function. Compute the limits, justify.

$$\lim_{\epsilon\to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}, \quad \lim_{\epsilon\to 0} \frac{f(x+\epsilon) - f(x-\epsilon,)}{\epsilon}, \tag{2}$$

3. $f(x)$ is a twice differentiable function. Compute the limit, justify.

$$\frac{f(x+\epsilon) - 2f(x) + f(x-\epsilon)}{\epsilon^2}, \tag{3}$$

4. Evaluate

$$\int_0^\pi \sin(2x)\,dx; \quad \int_0^1 \cos(kx)\,dx \quad \int_0^1 \exp(kx)\,dx \tag{4}$$

5. Evaluate

$$\int_0^a x\sin(kx)\,dx; \quad \int_0^a x^2\sin(kx)\,dx \quad \int_0^a \exp(cx)\sin(kx)\,dx \tag{5}$$

6. What is a basis of a vector space? Now many vectors form a basis for a three-dimensional space.

7. Represent a vector $(1,2)$ is the basis $\mathbf{a_1}$, $\mathbf{a_2}$, where $\mathbf{a_1} = (1,-1)$ and $\mathbf{a_2} = (1,2)$

8. Define a scalar product of two vectors.
9. Compute $\mathbf{a} \cdot \mathbf{b}$, where $\mathbf{a} = (1, 1, 1)$ and $\mathbf{b} = (-1, 3, 0)$
10. Define x so that vectors $\mathbf{p}$, $\mathbf{q}$ became orthogonal, where $\mathbf{p} = (1, 2, 3)$ and $\mathbf{q} = (1, -1, x)$
11. What is an eigenvector of a matrix?
12. Find eigenvectors and eigenvalues for the matrix

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \tag{6}$$

13. What is a particular and a general solution of an ODE?
14. Solve

$$u'' + \omega^2 u = \cos(kt), \quad u(0) = 0, u'(0) = 1 \tag{7}$$

15. Solve

$$u' + \alpha u = 2\exp(kt), \quad u(0) = 1 \tag{8}$$