CMDS13 Conference Abstracts
[alphabetical by author name]

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1. **Nathan Albin,** albin@math.ksu.edu
   Modulus of Path Families: Continuous and Discrete

   The modulus of a family of paths in a metric space provides a quantitative assessment of the "richness" of the family: large families of short paths have larger modulus than small families of long paths. In Euclidean space, the concept of modulus is closely related to the concept of capacity and to certain energy minimization problems over gradient vector fields. In the discrete setting, the concept of modulus can be linked to several graph-theoretic quantities including shortest path, minimum cut, and effective resistance. This talk will present connections among these concepts, suggest possible applications, and introduce a new numerical algorithm for computing the modulus of path families on discrete grids and on graphs in general.

2. **Andrea Alu,** alu@mail.utexas.edu
Co-authors: Dimitrios Sounas, Romain Fleury
Parity-time symmetric metasurfaces and metamaterials

Metamaterials are artificially structured materials possessing exotic electromagnetic or acoustic properties that are not readily available in nature, for instance synthesizing negative, zero, or very large index of refraction. Their exotic features are typically associated with narrow bandwidths and losses. In this talk, we introduce the concept and theoretical modeling of parity-time (PT) symmetric metasurface pairs and metamaterials, which offer a unique solution to these issues, realizing loss-compensated, broadband wave manipulation, including negative refraction, cloaking and planar focusing. We discuss the largely uncharted scattering properties of PT-symmetric metamaterials and metasurfaces, with an emphasis on the exciting mathematical and physical aspects involved in these problems.

3. **Ali Rana Atilgan,** `atilgan@sabanciuniv.edu`
Co-authors: Canan Atilgan
Multiscale Interactions and Dynamical Systems Theme

Non-synonymous single nucleotide polymorphisms within the coding regions of genes giving rise to amino acid substitutions have shown to alter drastically biological function of proteins. We have developed methods to simply, yet efficiently, characterize the effects of those substitutions on the conformational changes and, therefore, on the dynamics and function of biological macromolecules [Annual Review of Biophysics, 41, 205 (2012)]. By changing the residue's physical–chemical characteristics, such as its charge distribution [Journal of Physical Chemistry B, 116, 7145 (2012)], or playing with the residue and its neighboring micro-environment [Journal of Chemical Physics, 135, 155102 (2011)], or altering the temperature of the surrounding water body [Journal of Chemical Physics, 133, 85101 (2010)], we have effectively demonstrated and confirmed with experimental studies that the turnover rate of an enzyme, or the time scale of ligand release may be altered in a controlled manner.

Bacteria continuously adapt to their environment, and while doing that, they carry mutations that take place at critical locations to their function. Random mutations taking place at the gene level may be expressed on proteins that are vital for the survival of the organism; with natural selection ("natural" referring to the current environment that may actually be extreme for normal life) these may become permanent. In this talk we formulate the ways to decipher the structural and dynamical changes induced on a given protein, by mutants that let the organism carry its normal function while conferring resistance, under the selection pressure induced by a given antibiotic.

4. **Katayun Barmak,** `kb2612@columbia.edu`
   Grain Growth and Grain Growth Stagnation in Metallic Films

   Polycrystalline metallic films play an important role in the advance of many modern technologies such as integrated circuits, information storage systems, displays, sensors and coatings. In integrated circuit technology, the transistors on the semiconductor chip are interconnected via narrow, thin metallic lines. Metallic films are also used as the magnetic information storage layer in hard disk drives, and as antenna and reflector elements in in optical applications. The physical behavior of metals in thin film form is often remarkably different from that of the corresponding bulk metallic material so it is important to study their structure and properties. The polycrystalline structure of the films forms during deposition and evolves during post-deposition processing via, for example, grain coarsening, more commonly termed grain growth. This talk focuses on the comparison of experiments and simulations of grain growth in metallic films. Experimental grain growth characteristics are examined by bringing together a large body of grain size data for Al and Cu films, in addition to data for the distribution of grain sides and other topology and topology-geometry metrics. The experimental data is used to evidence stagnation of grain growth and to arrive at a universal experimental grain size distribution. This universal distribution deviates from the distribution obtained in two-dimensional simulations of grain growth with isotropic boundary energy in two notable regions termed the "ear" and the "tail". In seeking the cause of the observed differences between experiment and simulation, the impact of surface and elastic strain energy, anisotropy of grain boundary energy, grain boundary grooving, impurity drag, and triple junction drag is examined. Recent work on comparison of experiments and simulations of the phase field crystal model of grain growth will also be discussed.

5. **Yakov Benveniste,** benben@eng.tau.ac.il
Exact results for the local fields and the effective moduli of fibrous composites with thickly coated fibers

Fibrous composites that consist of thickly coated cylindrical fibers embedded in a matrix are considered [1]. All of the three phases of the composite are assumed to be transversely isotropic. The study consists of three parts. In the first part exact relations are derived for the influence functions that connect applied uniform overall fields to the induced local fields in piezomagnetoelectric systems. We consider the case of coated fibers with a concentrically circular cross section, and contrast the derived relations with the more limited ones that could be obtained in the case of coated fiberswith an arbitrary cross section. The derivation is based on the ability to create uniform strain, electric and magnetic fields in the composite by the application of certain mechanical, electric, magnetic and thermal loadings. In the second part of the study, exact microstructure independent connections are derived for a subgroup of the effective moduli of the homogenized piezomagnetoelectric composite which exhibits overall transverse isotropy. The first and second parts become a generalization of some of the contents of [2] to the above described coated-fiber composite. In the third part of the study, the derived exact connections between the effective moduli are reduced to the setting of thermoelasticity; letting the coating to be thin and highly stiff or highly compliant, we make contact with the exact connections derived in [3] and [4] for two-phase fibrous thermoelastic composites with surface-stress-type and spring-type imperfect interfaces.

References

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6. **Viktor Berdichevsky,** vberd@eng.wayne.edu
Homogenization of random structures in probabilistic terms

Studying of materials with evolving random microstructures requires the knowledge of probabilistic characteristics of local fields because the path of the microstructure evolution is determined by the local fields. The mathematical problem for determining the probabilistic characteristic was formulated as a minimization problem for energy with the respect to probabilistic measure long ago (Berdichevsky, 1987). This variational problem contains an infinite chain of constraints for probability distributions. It remained unclear how to properly truncate this infinite chain. In this talk a way of truncation is suggested. The corresponding probability distributions are found for two-phase composites. As a by-product, one gets also the dependencies of the effective coefficients on the parameters of microstructures. Other issues that will be discussed are: formulation of Hashin–Shtrikman results in probabilistic terms, a universal extension of the Hashin–Shtrikman bounds, the law of decrease of microstructure entropy.

7. **David J. Bergman,** bergman@post.tau.ac.il
Co-authors: Asaf Farhi
Exact analysis of a Veselago lens in the quasi-static regime

The resolution of conventional optical lenses is limited by the wavelength. Materials with negative refractive index have been shown to enable the generation of an enhanced resolution image where both propagating and non-propagating waves are employed. We analyze such a Veselago lens by exploiting some exact one dimensional integral expressions for the quasi-static electric potential of a point charge in that system. Those were recently obtained by expanding that potential in the quasi-static eigenfunctions of a three-flat-slabs composite structure. Numerical evaluations of those integrals, using realistic values for physical parameters like the electric permittivities of the constituent slabs and their thicknesses, reveal some surprising effects: E.g., the maximum concentration of the electric field occurs not at the geometric optics foci but at the interfaces between the negative permittivity slab and the positive permittivity slabs. The analysis provides simple computational guides for designing such structures in order to achieve enhanced resolution of an optical image.

8. **Liliana Borcea,** `borcea@umich.edu`
   Imaging in complex environments

   The talk is concerned with the application of sensor array imaging in complex environments. The goal of imaging is to estimate the support of remote sources or strong reflectors using time resolved measurements of waves at a collection of sensors (the array). This is a challenging problem when the imaging environment is complex, due to numerous small scale inhomogeneities and/or rough boundaries that scatter the waves. Mathematically we model such complexity (which is necessarily uncertain in applications) using random processes, and thus study imaging in random media. I will focus attention on the application of imaging in random waveguides, which exhibits all the challenges of imaging in random media. I will present a quantitative study of cumulative scattering effects in such waveguides and then explain how we can use such a study to design high fidelity imaging methods.

9. **Rebecca Brannon,** `Rebecca.Brannon@utah.edu`
Co-authors: Farah Huq, Lori Graham-Brady
Initialization of Discrete Statistical Crack Properties in Continuum-Scale Domains

Aleatory uncertainty in continuum damage models is a direct result of variability in the number, size, and orientation of flaws within a finite-element. Each discrete flaw in a structural ceramic may be modeled explicitly only if the finite element is extremely small ($<$ 0.1mm across). Beyond a few millimeters (as needed in engineering applications), homogenization of discrete expectation summations is needed to estimate ensemble effects of many flaws. It is not computationally tractable to evaluate expectation summations over each of the millions or trillions of flaws in these larger elements, but they still are not large enough to justify replacing the discrete flaw realizations with a continuous approximation. In fact, it is proved that the finite elements would need to be of "planetary" size in order for the discrepancy between a discrete and continuous expectation to fall below numerical round-off. To address this difficulty, mapping principles of the Material Point Method are applied to obtain a novel and flexible flaw binning scheme. At initialization, the original Poisson-process realization of the number of cracks is mapped to a background grid, and then discarded. The Dirac deltas in the original (exact) discrete joint PDF for crack size and orientation are thus mapped (in probability reference space) to a grid-based continuous delta-like approximate field. Gauss locations and weights for averages over this field provide bin properties, after which the grid is also discarded, leaving only a much smaller set of equivalent cracks for subsequent crack evolution calculations. The approach rigorously converges to the original discrete PDF as the grid is refined, and it easily handles any number of random variables for each flaw (size, orientation, planarity, etc.). Judicious grid zooming to favor larger cracks (because of their greater influence on stiffness and strength) provides high accuracy of the binned approximation of the original expectation integrals at a fraction of the computational cost.

10. **Nathan Briggs,** nathancbriggs@gmail.com
A Look at the Numerical Results of the Optimal Design of Multiphase Elastic Structures

The optimal design problem is considered: It is required to lay out two elastic materials and void in the design domain in order to minimize the elastic energy stored in the domain plus the total cost. It is assumed that the cost of each material is known: the strong material is expensive, the weak material is cheap. The loading and supports are fixed. The problem of optimal three material composite as formulated and solved in [1], optimal microstructures were defined for various loading conditions. Mathematically, this problem is a multivariable nonconvex minimization problem: determination of the quasiconvex envelope of a multiwell Lagrangian, where the wells represent materials' energies plus their costs; the quasiconvex envelope represents the energy and the cost of an optimal composite [2]. The main focus of the talk is to investigate the problem of finding the optimal design of a macrostructure consisting of the previously discussed microstructures. The compliance of a macrostructure is minimized with respect to both microstructure and volume fraction for a given state of stress, several examples are computed and discussed. The found optimal structures explain the role of each material in the optimal design. The results are compared with two-material optimal designs.

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[2] A. Cherkaev. Variational Methods for Structural Optimization. Springer

[3] N. Briggs, A. Cherkaev and G. Dzierzanowski. 2013. Note of Optimal design from three materials. Submitted to Int. J. of Structural Optimization.

11. **Michele Brun,** mbrun@unica.it
Co-authors: Daniel Colquitt, Massimiliano Gei, Alexander B. Movchan, Natasha V. Movchan, Ian S. Jones
Transformation cloaking for flexural waves in elastic plates

Cloaking transformation for flexural waves in plates are considered. For the Kirchhoff-Love problem governed by a fourth-order partial differential equation it is shown that the equation governing the problem is not invariant with respect to cloaking transformation as opposed to the Helmholtz operator. The transformed equation finds a consistent physical interpretation as a inhomogeneous orthotropic flexural plates in the presence of pre-stress and in-plane external forces. Additionally, an asymptotic derivation from three-dimensional continuum elasticity provides a rigorous link between the model in question and elastic wave propagation in thin solids, showing connection with non-symmetric formulation in vector elasticity studied in earlier works. Example on a regularized push-out transformation show the broadband cloaking effect within the plate Kirchhoff-Love plate theory. The proposed physical interpretation of the transformed equation may lead to a refinement of the experimental implementations.

12. **Blaise Bourdin,** bourdin@lsu.edu
Multiscale Problems in Brittle Fracture

The Variational Approach to Fracture has now emerged as a versatile yet rigorous model of fracture in brittle materials. Its natural ability to handle crack propagation along complex unknown geometry in two and three dimensions is allowing unprecedented numerical investigations of problems involving very complex fracture systems. I will focus on two such situations: the morphogenesis of complex cracks induced by thermal shocks, and fracture of heterogeneous materials. In the first situation, we will show how the regularized fracture energies used in the numerical simulations can be seen as ad-hoc graduent damage models in order to faithfully reproduce crack nucleation then propagation over many order of magnitude in scale. In the later one, we will show how the intuitive concept of "effective toughness" of an heterogeneous material may be inappropriate. We present a new formalism based on a "surfing experiment" for the determination and numerical computation of this effective toughness.

13. **Xian Chen,** chen1561@umn.edu
Co-authors: Yintao Song, Richard D. James
From compatibility conditions to stress-free microstructure for martensitic transformation The cofactor conditions (CC) are the conditions of super compatibility between phases for martensitic transformation. By satisfying CC, austenite and variants of martensite can fit together without elastic transition layers for any twinning volume fraction between 0 and 1. Here we discuss different forms of CC in Type I/II, Compound twins and domains, followed by the prediction of their possible microstructures. Then we calculate the geometric linear case of CC. Finally, we show real examples whose lattice parameters were tuned to satisfy CC closely for both Type I and II twin system.

Materials undergoing reversible phase transformations are desirable for applications in medical devices, microelectronics and energy conversion devices [1]. The conditions of elastic compatibility of the phase-transforming materials have profound impact on these applications. Because they dominate the hysteresis, the reversibility and the resistance to functional degradation upon cyclic operation [2, 3]. It has been proven experimentally that the degree of hysteresis and the cycling ability of the materials can be optimized [4] by making the lattice parameters satisfy a set of geometric compatibility conditions

$$\lambda_2 = 1 \tag{1}$$

$$\mathbf{a} \cdot \mathbf{U}\mathrm{cof}(\mathbf{U}^2 - \mathbf{I})\mathbf{n} = 0 \tag{2}$$

$$\mathrm{tr}\mathbf{U}^2 - \det \mathbf{U}^2 - \tfrac{|\mathbf{a}||\mathbf{n}|}{4} \geq 2 \tag{3}$$

where λ_2 is the middle eigenvalue of the transformation stretch tensor $\mathbf{U}$, the vector $\mathbf{a}$, $\mathbf{n}$ are the twinning parameters. Microscopically, the satisfaction of these conditions means a perfect lattice points match at the interface between the initial and final phase, which results in stress-free microstructure in the material. Therefore, it allows for materials transforming back and forth freely without paying internal elastic penalty. This is why the materials satisfying CC exhibit tremendous reversibility.

14. **Andrej Cherkaev,** cherk@math.utah.edu
Optimal multimaterial composites: Bounds and structures

The paper suggests a method for funding exact bounds for the effective conductivity moduli of multimaterial composites. These bounds expand and refine Hashin–Shtrikman and Nesi bounds. We prove that the fields in the materials within optimal structures vary in restricted domains and take this into account, obtaining more restricted bounds. The new bounds are solutions of a formulated relaxed finite-dimensional constrained optimization problem. For two-dimensional conducting three-material composites, bounds for effective conductivity are explicitly computed. These bounds are exact: Three-material isotropic microstructures of extremal conductivity are found that realize the bounds for all values of parameters. The optimal structures are laminates of a finite rank, their parameters vary with the volume fractions and they experience two topological transitions: For large values of material of minimal conductivity, its subdomain percolates (is connected), for intermediate values of that fraction, no material forms a connected domain, and for small values of that fraction, the domain of intermediate material percolates. Another type of isotropic optimal three–material structures is the "wheel assemblages" that replaces the Hashin–Shtrikman coated circles.

15. **Elena Cherkaev,** elena@math.utah.edu
Spectral representation in forward and inverse homogenization problems for composite materials

The talk discusses spectral representation of the effective properties of composites and inverse homogenization problem which is a problem of deriving information about the microgeometry of the composite from given effective properties. The approach is based on the spectral measure of a self-adjoint operator that depends on the geometry of composite. Stieltjes analytic representation of the effective property relates the n-point correlation functions of the microstructure to the moments of the spectral measure, which contains all information about the microgeometry. I show that the problem of identification of the spectral function from effective measurements known in an interval of frequency, has a unique solution. In particular, the volume fractions of materials in the composite and an inclusion separation parameter, as well as the spectral gaps at the ends of the spectral interval, can be uniquely recovered. I will discuss reconstruction of microstructural parameters from electromagnetic and viscoelastic effective measurements, application to coupling of different effective properties, and show an extension to nonlinear and multi-component composites.

16. **Alexander Chudnovsky,** achudnov@uic.edu
Scale Effect in Brittle Fracture

Heterogeneous morphology and a hierarchy of defects from grain size level up to a large-scale network of cracks are typical for most of engineering materials. It create a challenge for determining macroscopic properties of materials, particularly the scale dependent ones such as strength, toughness and other brittle fracture characteristics. The most pictorial manifestation of material heterogeneity are highly irregular and random fracture surfaces. Modeling of brittle fracture of micro-heterogeneous materials calls for a synthesis of fracture mechanics with probability and stochastic calculus. This work present a brief overview of Statistical Fracture Mechanics that addresses the statistical aspects of fracture. It is illustrated by studies of concrete brittle fracture and includes determination of a scaling rule for fracture toughness. Complex interaction between a macroscopic crack and an array of pre-existing micro-defects results in highly tortuous, stochastic fracture paths and in large scatter of all fracture-related parameters. Results of application of Statistical Fracture mechanics and related experimental studies are reported in this presentation.

17. **Daniel Colquitt,** `d.colquitt@imperial.ac.uk`
Co-authors: RV Craster, T Antonakakis, S Guenneau
In-plane Rayleigh-Bloch waves for elastic diffraction gratings

Despite being extensively studied in scalar wave systems, Rayleigh–Bloch waves in elasticity appear to have received very little attention. Given the significance of Rayleigh–Bloch waves in applications and the pervasive nature of diffraction gratings, there has been no investigation of whether such waves occurs within elastic diffraction gratings for the in-plane vector elastic system. In the present work, we demonstrate that Rayleigh–Bloch waves do exist for elastic diffraction gratings and identify the boundary conditions that support such waves. In parallel, an asymptotic procedure is developed that allows the diffraction grating to be replaced by an homogenised continuum at all frequencies, whilst retaining the essential features of the Rayleigh–Bloch waves. Numerical simulations demonstrate the efficacy of the asymptotic scheme.

18. **Shankar Prasad Das,** shankar0359@yahoo.com
Co-authors: Nita Bidhoodi, Madhu Priya
Qualitatively different collective and single particle dynamics in a supercooled liquid

The dense supercooled liquid develops finite shear modulus below a characteristic temperature though crystallization of the system in to a structurally ordered state has not occurred. The process of formation of the amorphous solid or the mechanism of glass formation remains an unsolved problem. In this work we demonstrate that the nature of the dynamics of a single tagged particle in the supercooled liquid is qualitatively different, having finite self diffusion constant even when the system has developed solid like properties at macroscopic length scales. Our analysis is based on the microscopic model of mode coupling theory (MCT) used for understanding slow dynamics in liquids. In its simplest form MCT predicts an ergodicity-nonergodicity (ENE) transition at a critical density n_c. The correlation function $\phi(q,t)$ of collective density fluctuations at wave vector q and time separation t is $f(q) \neq 0$ at the transition. We show that maintaining consistency with the basic conservation laws of the system requires that even beyond the ENE transition, the self-diffusion constant D_s of a tagged particle remains finite. Only if the mode coupling effects are evaluated in the so called adiabatic approximation which violates momentum conservation, D_s goes to zero at the ENE transition.

19. **Grzegorz Dzierzanowski,** gd@il.pw.edu.pl
Co-authors: Nathan Briggs, Andrej Cherkaev
Optimal structures made of two elastic materials and void

In this research we use the framework of theory of elasticity to describe the exact solution in optimal design of three-phase structures of minimal compliance, or equivalently, maximal stiffness. Our problem is formulated as follows: lay out two elastic materials and void in a given two-dimensional design domain Ω, loaded at its boundary $\partial\Omega$, so that the structural compliance achieves its minimal value. It turns out that optimal layout of phases is determined on two levels: macro and microscopic. On the macrolevel, the design domain is divided into subdomains filled with pure phases or their mixtures (composites) with optimal microgeometries. The main aim of the talk is to discuss the case when optimal macroscopic distribution of phases is not unique.

20. **Yekaterina Epshteyn,** `epshteyn@math.utah.edu`
Co-authors: K. Barmak, E. Eggeling, M. Emelianenko, D.Kinderlehrer, R. Sharp, S. Ta'asan and P. Bardsley
A Theory and Challenges for Coarsening in Microstructure

Cellular networks are ubiquitous in nature. Most technologically useful materials arise as polycrystalline microstructures, composed of a myriad of small crystallites, or grains, separated by interfaces, or grain boundaries. The energetics and connectivity of the grain boundaries network plays a crucial role in determining the properties of a material across a wide range of scales. Coarsening, or growth process is influenced mainly by the effort of the system to decrease the interfacial energy subject to spatial constraints. The recently discovered Grain Boundary Character Distribution (GBCD) indicates that the boundary network of a cellular structure, and, more generally, material texture has a natural order. In this talk we will present and discuss a theory for the evolution of this statistic which leads to many interesting and challenging questions.

21. **Alexander Figotin,** afigotin@uci.edu
Co-authors: Guillermo Reyes
Multi-transmission-line-beam interactive system

We construct a Lagrangian field formulation for a system consisting of an electron beam interacting with a slow-wave structure modeled by a possibly non-uniform multiple transmission line (MTL). In the case of a single line we recover the linear model of a traveling wave tube (TWT) due to J.R. Pierce. Since a properly chosen MTL can approximate a real waveguide structure with any desired accuracy, the proposed model can be used in particular for design optimization. Furthermore, the Lagrangian formulation provides for: (i) a clear identification of the mathematical source of amplification, (ii) exact expressions for the conserved energy and its flux distributions obtained from the Noether theorem. In the case of uniform MTLs we carry out an exhaustive analysis of eigenmodes and find sharp conditions on the parameters of the system to provide for amplifying regimes.

22. **Alexander Freidin,** alexander.freidin@gmail.com
Co-authors: Elena Vilchevskaya
Chemical affinity tensor in mechanochemistry of deformable solids

Chemical affinity is one of the basic notions in physical chemistry since its value determines a chemical reaction rate. In the case of liquid and gas constituents the chemical affinity is a scalar equal to the combination of chemical potentials of the constituents. Recently it was shown that the chemical affinity is a tensor in the case of solid constituents. An expression of the chemical affinity tensor was derived as a consequence of the mass, momentum and energy balances and entropy inequality written down for an open system "solid–gas" in which a chemical reaction took place between gas and solid with arbitrary constitutive equations. In a quasi-static case the chemical affinity tensor takes the form of the combination of chemical potentials tensors of the solid constituents (equal to the Eshelby stress-tensors divided by the reference mass densities) and the chemical potential of the gas constituent. Having the expression of the chemical affinity tensor, we formulate a kinetic equation for the reaction rate at the oriented surface element in a form of the dependence on the normal component of the chemical affinity tensor. Then we study the role of the invariants of the chemical affinity tensor. We specify the expressions for the case of linear elastic solid constituents. We find the orientation of the surface elements at which the reaction rate is maximal using the formalism developed earlier for interfaces in the case of phase transformations. We formulate boundary value problems of mechanochemistry and study a planar chemical reaction front propagation and solve axially-symmetric and spherically-symmetric problems of mechanochemistry. Finally we consider inelastic solid constituents and incorporate the relaxation of stresses produced by chemical transformation strain.

23. **Joe Goddard,** jgoddard@ucsd.edu
Continuum modeling of granular media

This talk summarizes a recent survey [1] of the interesting phenomenology and the prominent régimes of granular flow which also offers a unified mathematical synthesis of continuum modeling. The unification is based on "parametric" viscoelasticity and hypoplasticity involving elastic and inelastic potentials. Fully non-linear, anisotropic visco-elastoplastic models are achieved by expressing the potentials as functions of the joint isotropic invariants of kinematic and structural tensors. These take on the role of evolutionary parameters or internal variables, whose evolution equations are derived from the internal balance of generalized forces. The resulting continuum models encompass most of the mechanical constitutive equations currently employed for granular media. Moreover, these models are readily modified to include Cosserat and other multipolar effects. Several outstanding questions are identified as to the contribution of parameter evolution to dissipation, the distinction between quasi-elastic and inelastic models of material instability, and the role of multipolar effects in material instability, dense rapid flow and particle migration phenomena.

[1] J. Goddard, *Appl. Mech. Rev.* **64**:5, 2014.

24. **Yury Grabovsky,** yury@temple.edu
A complete list of exact relations for effective elastic tensors of fiber-reinforced composites

Microstructure-independent relations for effective tensors of composites have been attracting the attention of materials scientists throughout the history of the subject. In the context of fiber-reinforced elastic composites, the confluence of fully 3-dimensional elastic tensors with an inherently 2D microstructure yields an enormous number of exact relations ranging from physically obvious to bewildering. Invariably, these relations can be represented by unexpectedly elegant formulas. I will describe a long and exciting journey starting from the general theory, through the efforts of Ph.D. and undergraduate students, to the complete list of exact relations.

25. **Fernando Guevera Vasquez,** `fguevara@math.utah.edu`
Co-authors: Liliana Borcea, Alexander V. Mamonov
A network based inversion method for the Schr`odinger problem

Consider the inverse problem of finding the Schr´odinger potential $q(x)$ from measurements of the Dirichlet to Neumann map of the Schr`odinger differential operator $u \to \Delta u + qu$. We present a method for solving this problem in 2D that is based on an analogous of the Liouville transform on resistor networks. The continuum Liouville transform maps an operator of the form $v \to \nabla \cdot [\sigma \nabla v]$, with $\sigma > 0$ to a Schr´odinger differential operator. The discrete Liouville transform on a network maps a Laplacian on the network to another Laplacian plus a diagonal perturbation which can be thought as a discrete Schr`odinger potential. The first step in the inversion is to find a resistor network that fits the data. In the second step, a discrete Liouville transform is applied to get a diagonal perturbation of a discrete Laplacian. The final step is to interpret the discrete Laplacian and its diagonal perturbation as a finite volumes discretization of the Schr´odinger differential operator. This gives a cheap reconstruction of the continuum Schr`odinger potential from the discrete one. The method needs to be calibrated by data for two known constant Schr´odinger potentials.

26. **Klaus Hackl, klaus.hackl@rub.de**
Co-authors: Muhammad Sabeel Khan
Modeling of microstructures in a Cosserat continuum using relaxed energies

A continuum model for granular materials exhibiting microstructures is presented using the concept of energy relaxation. In the framework of Cosserat continuum theory the free energy of the material is enriched with an interaction energy potential taking into account the counter rotations of the particles. The total energy thus becomes non-quasiconvex, giving rise to the development of microstructures. Relaxation theory is then applied to compute its exact quasiconvex envelope. It is worth mentioning that there are no further assumptions necessary here. The computed relaxed energy yields all possible field fluctuations of displacements and micro-rotations as minimizers. We show that the material behavior can be divided into three different regimes. Two of the material phases are exhibiting microstructures in rotational and translational motion of the particles, respectively, and the third one is corresponding to the case where there is no internal structure of the deformation field. The properties of the proposed model are demonstrated by carrying out numerical computations. The obtained results exhibit a number of unexpected features, for example the transition between distributed and localized microstructures.

27. **Yilong Han,** yilong@ust.hk
Co-authors: Yi Peng, Feng Wang
Nucleation in solid-solid transitions of colloidal crystals

Solid-solid phase transitions between different crystalline structures are ubiquitous in nature, but their kinetic pathways and mechanisms present formidable challenges for theory, simulation and experiment. Here we directly imaged the solid-solid transitions in colloidal thin films composed of diameter-tunable NIPAM microspheres with single-particle resolution by video microscopy. We discover a surprising two-step diffusive nucleation behavior for transitions from square- to triangular-lattices with an intermediate liquid stage. The observations and resulting theoretical analysis suggest that, provided solid-liquid interfacial energies are sufficiently small, s-s transitions in most traditional metals and alloys should follow this two-step nucleation with intermediate liquid stage, and should generally arise in 2D, 3D and thin-film single crystals and polycrystals. The nucleation precursors are particle-swapping loops rather than structural defects, which, in turn, provide a new relaxation mode that makes s-s transitions easier and faster. This new kinetic factor controlling the s-s transition rate has never been considered and should be incorporated in future s-s transition theories.

Applying a small anisotropic strain can reduces the liquid nucleus size. Above a threshold of the applied strain, the intermediate liquid nuclei vanished. Instead, a few pairs of dislocations were first generated from the square lattices as nucleation precursors, which triggered tens of particles to collectively transform to a triangular-lattice nucleus and then grew diffusively. This martensitic transformation at the early stage and the diffusive nucleation at the later stage is another novel type of kinetic pathway in solid-solid transition.

In addition, we observed that the coherent and incoherent facets of the evolving nuclei exhibit different energies and growth rates which can dramatically alter nucleation kinetics. The coalescence of two crystalline nuclei exhibits different behaviors for different lattice angles.

28. **Alex Hansen, Alex.**`Hansen@ntnu.no`
Crack front propagation using the fiber bundle model

Disentangling the dynamics of a crack moving along a weak plane is along–standing problem that has interested the statistical physics community for nearly twenty years. The advancing crack front has been shown experimentally to exhibit all the signatures of a dynamical critical phenomenon. The theoretical approach to this problem has been a top–down one where one uses LEFM as a starting point. We take here an opposite view and approach the problem bottom-up. By using the fiber bundle model, we are able to connect with the LEFM–based approach explaining a number of open question on the way.

29. **Davit Harutyunyan,** davith@math.utah.edu
Co-authors: Graeme W.Milton
Examples of extremal quasiconvex quadratic forms that are not polyconvex

We prove that if the associated fourth order tensor of a quadratic form has a linear elastic cubic symmetry then it is a quasiconvex form if and only if it is polyconvex, i.e., a sum of convex and null–Lagrangian quadratic forms. We prove that allowing for slightly less symmetry, namely only cyclic and axis–reflection symmetry, gives rise to a class of extremal quasiconvex quadratic forms, that also turn out to be non–polyconvex.

30. **Hisao Hayakawa,** hisao@yukawa.kyoto-u.ac.jp
Co-authors: Koshiro Suzuki
Theory on rheology of dense granular flow

In this talk, I will discuss on possible theoretical approaches on dense granular flows. The systems I will discuss are both plane shear and pulling a tracer particle in a granular medium. The main part of this talk will be focused on the first topic. In the first part, I will summarize some identities of granular flows, known as the fluctuation theorem and the generalized Green-Kubo formula. These identities are basis of the analysis of granular flows. In the second part, I will explain the formulation to describe the granular flows, where I have adopted the mode coupling theory (MCT) to get a closure. In the third part, I apply MCT to the sheared granular flow. Thanks to the coupling between the density correlation function and the cross correlation function of density-current, the plateau of the density correlation function is suppressed. MCT also can predict the viscosity if the density shift is adopted. Nevertheless, MCT cannot be used to describe the jamming transition as it is. I will also discuss the possible extension of MCT how to describe the jamming transition. In the fourth part, I apply MCT to the drag problem of a pulling tracer particle in a granular medium. This can be regarded as one of problems on microrheology.

31. **Gengkai Hu,** hugeng@bit.edu.cn
Co-authors: Xiaoning Liu, Yi Chen, Guoliang Huang
Elastic metamaterials with chiral microstructure

By carefully designing microstructures of composites to promote internal resonances under wave loading, elastic metamaterials with negative material parameters (density, bulk and shear modulus) can be obtained over a certain frequency. This property may have a great potential to control low-frequency elastic waves. In the first part of the talk, we will explain how to realize elastic metamaterials with chiral microstructure to trigger rotational resonance. Chiral lattice with embedded resonators and thin plate with chiral microstructure are designed and the both are demonstrated to have simultaneous negative effective mass density and bulk modulus over certain frequency, negative refraction of the latter is also verified experimentally by a transient elastic wave tests. In the second part of the talk, we will present a homogenization method for planar chiral lattices such as tetrachiral lattices (isotropic chiral lattice as a special case), a bi–dimensional orthotropic chiral micropolar model is developed based on the theory of irreducible orthogonal tensor decomposition. The material constants of the continuum model are analytically derived by a homogenization process. By comparing with discrete lattices, the proposed continuum model is shown to be able to characterize correctly the wave properties of the chiral lattices.

32. **Lee Hyundae,** hdlee@inha.ac.kr
Co-authors: Hyeonbae Kang, Kyoungsun Kim, Xiaofei Li, Graeme W. Milton
Size estimate in complex conductivity using translation method

The size estimation problem in electrical impedance tomography is considered when the conductivity is a complex number and the body is two-dimensional. Upper and lower bounds on the volume fraction of the unknown inclusion embedded in the body are derived in terms of two pairs of voltage and current data measured on the boundary of the body. These bounds are derived using the translation method. We then show by numerical examples that these bounds are quite tight and stable under measurement noise.

33. **Dominique Jeulin, dominique.**`jeulin@mines-paristech.fr`
On the statistical RVE of some long range random media

It is usual to estimate the variance of local averages by means of the integral range (or integral of the centered normalized correlation function). This procedure is followed to estimate the representative volume elements (RVE) of the microstructure for numerical homogenization of effective properties on simulations. Long fibers or stratified media show very long range correlations, inducing an infinite integral range. This media can be simulated by models of Boolean random varieties. For these models non standard scaling power laws exist for the decrease of the variance of local averages, like the volume fraction, with the volume of domains K . In R^n, the asymptotic expression of the variance $D^2_Z(K)$ of the local fraction $Z = \frac{\mu_n(A \cap K)}{\mu_n(K)}$ ($\mu_n(K)$ being the Lebesgue measure of K) of a Boolean model built on isotropic Poisson varieties of dimension k ($k = 0, 1, ..., n-1$) V_k, is expressed by [1]:

$$D^2_Z(K) = p(1-p)\left(\frac{A_k}{\mu_n(K)}\right)^{\frac{n-k}{n}}$$

The exponent $\gamma = \frac{n-k}{n}$ is equal to $\frac{2}{3}$ for Boolean fibers in 3D, and $\frac{1}{3}$ for Boolean strata in 3D. When working in 2D, the scaling exponent of Boolean fibers is equal to $\frac{1}{2}$. This reflects a much lower decrease of fluctuations with respect to the volume of observations, as compared to media with a finite integral range, where $\gamma = 1$. As a result, very large domains have to be simulated to obtain statistical RVE of the microstructure for numerical homogenization. This behaviour was checked in the case of the estimation of elastic moduli and thermal conductivity of simulations of various fiber systems.

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34. **Ken Karmin,** `kkamrin@mit.edu`
Co-authors: David Henann, Georg Koval
Nonlocal continuum modeling of dry granular flow

Despite the ubiquity of granular matter in the world around us, the challenge of predicting the motion of a collection of flowing grains has proven to be a difficult one, from both computational and theoretical perspectives. In this talk, we begin by presenting a number of the "unusual" behaviors exhibited by dry granular media, which have posed hurdles from the perspective of developing a continuum model. These behaviors include: steady-flow fields that do not obey any local flow rheology, flow onset and stoppage phenomena that do not abide by a standard yield stress, and the motion-induced "quicksand" effect whereby far-away motion changes the flow resistance everywhere. Herein we present a non-local constitutive relation for granular matter, and demonstrate that it correctly captures each of these unusual phenomena. This is achieved by corroborating the model's predictions against hundreds of existing experimental data sets, which elucidate the aforementioned behaviors.

35. **Robert V. Kohn,** kohn@cims.nyu.edu
Co-authors: Benedikt Wirth
Optimal design revisited: The leading order effect of perimeter regularization

We consider a 2D shape optimization problem involving a structure loaded in shear. When the goal is to minimize compliance, optimality requires microstructure and a 2nd rank laminate is known to suffice. We ask the question: what happens when the goal is to minimize compliance plus epsilon times perimeter? Our main accomplishment is to identify the leading-order effect of the perimeter regularization. This entails proving upper and lower bounds that match (with respect to scaling). The construction associated with the upper bound resembles a second-rank laminate, but uses branching rather than approximate interfaces. The ansatz-free lower bound uses the Hashin–Shtrikman variational principle, and therefore works mainly in Fourier space

36. **Michal Kordy,** kordy@math.utah.edu
Co-authors: Elena Cherkaev, Phil Wannamaker
Forward and inverse multi–frequency problem for Maxwell's equations using adaptive model order reduction

This work develops a model order reduction method for numerical solution of forward and inverse multi–frequency eddy current problem. Using Helmholz decomposition, we extend previously developed technique to the case when the operator has a non-empty null space. In the case of Finite Element discretization of Maxwell's equations with edge elements, the discrete Helmholtz decomposition is accomplished by solving a Poisson equation on nodal elements of the same grid. Exploiting analyticity of the electromagnetic field, we use Pade interpolation in the complex frequency plane; this allows us to approximate the forward solution as well as the frequency-dependent jacobian in the inversion procedure. To adaptively choose interpolating frequencies, we propose to minimize the maximal approximation error. We discuss several error estimates and propose a fast method of calculating the residual across a range of frequencies. The efficiency of the developed approach is demonstrated by applying it to the forward and inverse magnetotelluric problem, which is a geophysical electromagnetic remote sensing method used in mineral, geothermal, and groundwater exploration. Numerical tests show excellent performance of the proposed methods characterized by significant reduction of computational time without loss of accuracy.

37. **Slava Krylov,** vadis@eng.tau.ac.il
Co-authors: Stella Lulinsky, Bojan R. Ilic, Inbar Schneider
Dynamics of Arrays of Parametrically Excited Electrostatically Coupled Micro Oscillators

In this work we investigate collective dynamics of an array of mechanically and electrostatically coupled micro cantilevers interacting through electrostatic forces. The motivation of the work is twofold. On the one hand, arrays of individually addressed elements could be used as sensors and the knowledge of their properties is necessary for the implementation of these devices in applications. On the other hand, large arrays of micro oscillators interacting by typically nonlinear and often tunable electrostatic forces exhibit rich dynamic behavior. In this prospective micro technology can be viewed as a convenient and versatile platform for the theoretical and experimental exploration of this kind of systems. The device considered in this work is composed of two sets of partially interdigitated flexible cantilevers. The coupling between the adjacent beams is a combination of a mechanical coupling appearing due to the clamping compliances and the electrostatic interaction through voltage-dependent electrostatic force. In the framework of the reduced order model built using the Galerkin decomposition the array is considered as an assembly of single degree of freedom oscillators. The mechanical coupling matrix is extracted using the full scale finite element analysis of the array. Collective dynamics of the array under linear kinematic and parametric electrostatic excitation are investigated numerically and experimentally. In the long wave quasi continuum limit the dynamics of the system are described by a nonlinear Klein-Gordon type equation and the array is equivalent to a string resting on an elastic foundation and pulled by a nonlinear voltage-dependent force. We show that large amplitude collective vibrations of the array can be achieved using parametric excitation while the dynamic properties of the array such as the width of the propagation band as well as the modal patterns can be efficiently tuned by the applied voltage.

38. **Rod Lakes,** `lakes@engr.wisc.edu`
Extreme elastic, viscoelastic and piezoelectric properties in structured materials

Materials with extremely high, even singular values of physical properties, are developed. Materials with designed heterogeneity including inclusions of negative compressibility can exhibit extremely high values of viscoelastic damping approaching a singularity, high Young's modulus (even greater than that of diamond) or piezoelectric sensitivity. Such behavior exceeds the usual theoretical bounds. The reason is that assumptions made in deriving the bounds can be relaxed in certain materials and microstructures. We consider toughness in the context of Cosserat elasticity in which there are characteristic lengths as additional engineering elastic constants. There are a total of six independent elastic constants in an isotropic Cosserat solid. Experimental work discloses a variety of cellular and fibrous materials to exhibit such freedom, and the characteristic lengths have been measured. In selected isotropic cellular solids all six of the Cosserat elastic constants have been measured. Several of these constants have been verified by further experiments in geometries different from those used in the original measurements. Holographic studies show that strain can spill over into regions which are classically forbidden, specifically the corners of a square cross-section prism in torsion.

39. **Khanh Chau Le,** chau.le@rub.de
Dislocations and formation of microstructures in single crystals

The present paper develops nonlinear continuum dislocation theory (CDT) to describe formation of microstructure in ductile single crystals. The key quantity distinguishing this theory from the strain gradient plasticity is the dislocation density being expressed in terms of the gradient of the plastic slip. We propose the thermodynamic framework of the CDT in which the elastic strain and the dislocation density are chosen as state variables. We specify the energy and dissipation in terms of these state variables and derive the governing equations and boundary conditions from the corresponding variational principles. We apply CDT to three problems: i) formation of lamellar structure in TWIP-alloys by deformation twinning, ii) formation of subgrain mictrostructure in bent single crystal beam by the process of polygonization, iii) formation of shear bands in micropillars under compression. We show that these processes are energy driven and compare the obtained results with experimental data. The excellent agreement justifies CDT.

40. **Mikyoung Lim, mikyoung.**`lim@gmail.com`
Co-authors: Sanghyeon Yu
Asymptotics of the solution to the conductivity equation in the presence of adjacent circular inclusions

We consider the conductivity problem in the presence of adjacent circular inclusions having arbitrary constant conductivity. When two inclusions get closer and their conductivities degenerate to zero or infinity, the gradient of the solution can be arbitrary large. We characterize the gradient blow-up by deriving an explicit formula for the singular term of the solution in terms of the Lerch transcendent function. This derivation is valid for inclusions having arbitrary constant conductivity. We illustrate our results with numerical calculations.

41. **Liping Liu, liu.**liping@gmail.com
Attainability of Hashin–Shtrikman's bounds for multiphase composites

We address the attainability of the Hashin–Shtrikman (HS) bounds for multiphase composite materials. We demonstrate that the HS bounds are not always attainable and give new restrictions on the attainable HS bounds in terms of the conductivities and volume fractions of the constituent phases. New optimal microstructures are also constructed to attain the HS bounds. Combined together, these results allow for precise characterization of the set of effective properties for a wide range of composite materials.

42. **Konstantin Lurie,** klurie@wpi.edu
On Material Optimization in Continuum Dynamics

The talk is aimed to discuss reasonable statement of the typical problems of optimal material optimization in continuum dynamics. Most of such problems are originally ill-posed and therefore require relaxation. In statics, such relaxation is achieved due to a material scaling accompanied by homogenization. Unlike that, in dynamics relaxation may be achieved through the involvement of dynamic materials - material formations with properties variable in space and time. The paper introduces some scenarios for such relaxation; in the absence of any general existence theorems, such scenarios are prompted by a suitable physical insight. Homogenization applies only in some of these scenarios.

43. **Fred MacKintosh,** `fcmack@gmail.com`
"Elasticity on the edge of stability: soft matter inspired by the cell

Much like the bones in our bodies, the cytoskeleton consisting of stiff protein biopolymers determines the mechanical stability and response of cells. Unlike passive materials, however, living cells are kept far out of equilibrium by metabolic processes and energy-consuming molecular motors that generate forces to drive the machinery behind various cellular processes. Inspired by such networks, we describe recent theoretical and experimental advances in our understanding of fiber networks in vitro and in vivo. We show that these exhibit a unique state of highly responsive matter near the isostatic point first studied by Maxwell [1,2]. For fiber networks, this represents a marginal state of matter with exceptional mechanical properties, including a strongly nonlinear elastic response and zero-temperature critical behavior [3]. Moreover, the introduction of molecular motor activity can dramatically affect the stability of such systems [4,5].

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44. **Alexander Mamonov,** mamonov@gmail.com
Co-authors: Vladimir Druskin, Mikhail Zaslavsky
Krein–Gelfand–Levitan algorithm for inverse hyperbolic problems via spectrally matched finite-difference grids

We present a method for the numerical solution of inverse problems for coefficients of hyperbolic PDEs based on the spectrally matched finite-difference grids (a.k.a. Gaussian quadrature rules or optimal grids). The method is built around an algorithm for interpolation of the measured time domain data. Once an interpolant is obtained, it can be expressed in terms of Stieltjes continued fraction or its matrix generalization. The use of S-fraction coefficients in inversion is twofold. First, they can be used to reformulate the traditional optimization-based approaches to drastically improve the objective functional, which addresses issues such as the abundance of local minima and slow convergence. Second, the coefficients provide a way to obtain direct, non-iterative reconstructions on the spectrally matched grids. We supplement the theoretical considerations with the numerical results.

45. **Ross McPhedran,** `ross@physics.usyd.edu.au`
Co-authors: J. O'Neill, O. Selsil, A.B. Movchan and N.V. Movchan
A Platonic View of Active Cloaking

We consider active cloaking of flexural waves in a Kirchhoff plate, where the objective is to minimise the far-field scattering signature of a rigid, clamped inclusion. The method relies on placing control sources at small distances from the scatterer and choosing their strengths to eliminate propagating orders of the scattered wave, thus reconstructing the incident wave. We use six control sources to obtain an effective configuration for cloaking the circular inclusion. Finally, we successfully cloak an arbitrarily shaped scatterer by deriving a semi-analytical, asymptotic algorithm. For further details, see arXiv:1403.0816

46. **Graeme W. Milton,** milton@math.utah.edu
Complete characterization of the macroscopic deformations of periodic unimode metamaterials of rigid bars and pivots

A complete characterization is given of the possible macroscopic deformations of periodic nonlinear affine unimode metamaterials constructed from rigid bars and pivots. The materials are affine in the sense that their macroscopic deformations can only be affine deformations: on a local level the deformation may vary from cell to cell. Unimode means that macroscopically the material can only deform along a one dimensional trajectory in the six dimensional space of invariants describing the deformation (excluding translations and rotations). We show by explicit construction that any continuous trajectory is realizable to an arbitrarily high degree of approximation provided at all points along the trajectory the geometry does not collapse to a lower dimensional one. In particular, we present two and three dimensional dilational materials having an arbitrarily large flexibility window. These are perfect auxetic materials for which a dilation is the only easy mode of deformation. They are free to dilate to arbitrarily large strain with zero bulk modulus.

47. **Sebastien Motsch,** `smotsch@asu.edu`
Co-authors: Pierre Degond, Irene Gamba, Jeff Haack, Laurent Navoret
Emergence of macroscopic behavior in complex systems

In a human crowd or in a shoal of fish, thousands of individuals interact and form large scale structures. Although the interaction among individuals might be simple, the resulting dynamics is quite complex. Modeling is an essential tool to understand such dynamics. For instance, Agent-based models, also referred to as microscopic models, are widely developed to analyze various dynamics such as swarming and opinion formations. In this talk, we investigate the emergence of macroscopic behavior for such models. The challenge is to link "microscopic models" describing each agent with "macroscopic models" describing the evolution of a fluid. To achieve this transition, we present a novel approach based on kinetic theory and asymptotic analysis. Numerical simulations are also presented to illustrate the results.

48. **Benjamin N. Murphy,** `nbmurphy@math.uci.edu`
Random Matrix Universality, Spectral Measures, and Composite Media

We consider composite media with a broad range of scales, whose effective properties are important in materials science, biophysics, and climate modeling. Examples include random resistor networks, polycrystalline media, porous bone, the brine microstructure of sea ice, ocean eddies, melt ponds on the surface of Arctic sea ice, and the polar ice packs themselves. The analytic continuation method provides a rigorous approach to treating the effective properties of such systems. At the heart of this method is a random matrix which depends only on the composite geometry. In this lecture we will discuss computations of the spectral measures of this operator which yield effective properties, as well as statistical measures of its eigenvalues. In particular, the effective behavior of these systems often exhibits large changes associated with transitions in the connectedness or percolation properties of a particular phase. We demonstrate that an onset of connectedness gives rise to transitional behavior in the eigenvalue correlations of the random matrix, which is captured by a one-parameter universality class of random matrix ensembles. This, in turn, gives rise to transitional behavior in the spectral measures, leading to observed critical behavior in the bulk transport properties.

49. **Akio Nakahara,** nakahara@phys.ge.cst.nihon-u.ac.jp
Co-authors: Hiroshi Nakayama, Yousuke Matsuo, Ooshida Takeshi
Control of desiccation crack patterns using memory effects of paste and Faraday waves

A densely packed colloidal suspension, called paste, remembers the direction of horizontal vibration and flow it experienced, and the memory of such motion can be visualized as a morphology of crack patterns which appear when the paste is dried. When the paste remember the direction of the horizontal vibration, desiccation cracks run in the perpendicular to the direction of the initial vibration. On the other hand, when the paste remembers the direction of the flow, desiccation cracks run in the direction parallel to the flow direction. Even if we had found a method to control the direction of crack propagation using memory effect of paste, it was difficult to control the position where cracks will appear. Here we find a method to control the position of crack formation by using both the memory effects and the Faraday waves which will appear when the paste is vibrated vertically. The experiments show that cracks tend to appear in the node zones of the Faraday waves, where the paste is vibrated locally and strongly in horizontal direction. We also succeed in making square lattice crack patterns using square lattice Faraday waves.

50. **Hoai-Minh Nguyen,** hoai-minh.nguyen@epfl.ch
Cloaking and Superlensing using complementary media

Negative index material (NIM) is an artificial structure where the refractive index has a negative value over some frequency range. NIMs were first investigated theoretically by Veselago in 1964 and were innovated by Nicorovici et al. in 1994 and Pendry in 2000. The existence of such materials was confirmed by Shelby et al. in 2001. NIMs has many interesting applications such as cloaking suggested by Lai et al. in 2009, superlensing proposed by Veselago in 1964, Nicorovici et al. in 1994 and Pendry in 2000, and cloaking by anomalous localized resonance by Milton et al. in 2006. An important class of NIMs related to these phenomena are complementary media. In this talk, I will present the ideas of cloaking and superlensing using complementary media and discuss their rigorous proofs. If time permits, cloaking by anomalous localized resonance is also discussed.

51. **Andrew Norris,** `norris@rutgers.edu`
Static and dynamic elastic homogenization of periodic structures

The talk considers (i) static homogenization of lattice structures modeled by thin beam members and (ii) a procedure for dynamic homogenization of general periodic elastic media. The seemingly disparate topics are motivated by a need to understand the static and dynamic behavior of lattice structures which have been proposed as candidates for pentamode materials. Our main result for *static* properties of a lattice structure with coordination number Z is that the effective moduli can be expressed in Kelvin-like form $\mathbf{C} = \sum_{i,j=1}^{N} P_{ij}\mathbf{U}_i \otimes \mathbf{U}_j$ where $N = \frac{1}{2}Z(Z+1)$, $\mathbf{U}_i$ are second order tensors, and P_{ij} are elements of a $N \times N$ projection matrix $\mathbf{P}$ of rank $N - d$, in $d = 2$ or 3 dimensions. The N second order tensors $\{\mathbf{U}_i\}$ split into Z stretch dominated and $N - Z$ bending dominated elements. The latter contribute little to the stiffness in the limit of very thin members, in which case the elastic stiffness is, at most, of rank $Z - d$. $\mathbf{C}$ is rank one if $Z = d + 1$, corresponding to pentamode materials. Part (ii) describes a general procedure for defining and calculating *dynamic* effective properties of periodic media. The dynamic homogenization yields constitutive relations for the effective medium of the so-called Willis type which couples stress with velocity, and momentum to strain. It turns out that there is then a unique system of equations governing the effective field variables defined according to $h^{\text{eff}}(\mathbf{x}, t) = \langle h \rangle\, e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ for each field variable (displacement, stress, etc.) where $\langle h \rangle$ is the spatial average of the periodic part of h. The effective dynamic parameters yield the exact dispersion relation for Bloch waves. Implications of the dynamic homogenization scheme for lattice networks will be discussed and compared with the static effective moduli.

52. **Braxton Osting,** braxton@math.ucla.edu
Co-authors: Jeremy Marzuola
Concentrated density of states and the triangular lattice

Motivated by a problem in solar cell design, we consider a spectral optimization problem for a weighted graph Laplacian on a geometric configuration of points, where the edge weights are dependent on pairwise distances. We seek finite (infinite) configurations of points whose associated Laplacian has eigenvalues (density of states) concentrated at a particular value. The challenge of this problem stems from the geometric constraints on the point configuration. Preliminary results show that for the two-dimensional periodic problem, the triangular lattice is a robust optimizer of this and several other spectral properties. This is joint work with Jeremy Marzuola.

53. **Alexander Panchenko,** anpanchenko@gmail.com
Non-local continuum models of particle systems

The main question addressed in the talk is how to obtain continuum equations for spatial averages from the ODEs of classical particle dynamics. Balance equations for the average density, linear momentum, and energy were derived by Irving and Kirkwood, Noll, Hardy, Murdoch and others. These equations are exact, but not in closed form since fluxes are given as functions of particle positions and velocities. Evaluating the exact fluxes requires solving the full ODE system which can be prohibitively expensive. We present a closure approximation that leads to continuum models in the true sense of the word. The fluxes in these models are given by operators acting on the average density and velocity. The closure construction is based on the use of regularized deconvolution. In the discrete setting, characterized by a finite non-improvable resolution, the error associated with deconvolution closure can be further reduced by incorporating a priori knowledge of empirical statistics of fluctuations. At the end of the talk we briefly discuss connections with large eddy simulation and quasi-continuum method. Results of numerical experiments and partial error estimates are presented as well

54. **Mikhail Raikh,** `raikh@physics.utah.edu`
Co-authors: R. C. Roundy
Spin dynamics of a diffusively moving electron in a random hyperfine field

We study the dynamics, $\langle S_z(t)\rangle$, of the average spin of electron hopping over sites which host random hyperfine magnetic fields. If the typical waiting time for a hop is τ and the typical magnetic fields is b_{s_0}, then the typical spin-precession angle on a given site is $\delta\phi \sim b_0\tau \ll 1$. Then the Markovian theory predicts that the spin, initially oriented along the z-axis decays, on average, as $\langle S_z(t)\rangle = \exp(-t/\tau_s)$, where $\tau_s = 1/b_0^2\tau$ is the spin-relaxation time. We find that in low dimensions, $d = 1, 2$, the decay, $\langle S_z(t)\rangle$, is non-exponential at all times. The origin of the effect is that for $d = 1, 2$ a typical random-walk trajectory exhibits numerous self-intersections. Multiple visits of the carrier to the same site accelerates the relaxation since the corresponding partial rotations, $\delta\phi$, of spin during these visits add up. As a result, the Markovian description does not apply. For one-dimensional diffusion of electron over sites, the average, $\langle S_z(t)\rangle$, is the universal function of $t^{3/2}/\tau^{1/2}\tau_s$, so that the characteristic decay time is $\tau^{1/3}\tau_s^{2/3}$ is much shorter than τ_s. Moreover, when the random magnetic fields are located in the (x, y) plane, the decay of $\langle S_z(t)\rangle$ to zero is preceded by a *reversal* of $\langle S_z(t)\rangle$ to the value $\langle S_z\rangle = -0.16$ at intermediate times. We develop an analytical self-consistent description of the spin dynamics which explains this reversal. Another consequence of self-intersections of the random-walk trajectories is that, in all dimensions, the average, $\langle S_z(t)\rangle$, becomes sensitive to a weak external magnetic field directed along z. Our analytical predictions are complemented by the numerical simulations of $\langle S_z(t)\rangle$.

55. **Gregory Rodin,** gjrodin@gmail.com
Co-authors: George J. Weng
On reflected interactions in elastic solids containing inhomogeneities

Interactions in linear elastic solids containing inhomogeneities are examined using integral equations. Direct and reflected interactions are identified. Direct interactions occur simply because elastic fields emitted by inhomogeneities affect each other. Reflected interactions occur because elastic fields emitted by inhomogeneities are reflected by the specimen boundary back to the individual inhomogeneities. It is shown that the reflected interactions are of critical importance to analysis of representative volume elements. Further, the reflected interactions are expressed in simple terms, so that one can obtain explicit approximate expressions for the effective stiffness tensor for linear elastic solids containing ellipsoidal and non-ellipsoidal inhomogeneities. For ellipsoidal inhomogeneities, the new approximation is closely related to that of Mori and Tanaka. In general, the new approximation can be used to recover Ponte-Castañeda-Willis' and Kanaun-Levin's approximations. Connections with Maxwell's approximation are established.

56. **Massimo Ruzzene,** ruzzene@gatech.edu
Co-authors: Marshall Schaeffer
Wave Propagation in Multistable Magneto-Elastic Lattices

The paper discusses the wave propagation characteristics of two-dimensional (2D) magneto-elastic meta-structures. These structures demonstrate the ability to undergo large topological and stiffness changes, which allows for dramatic changes in wave propagation characteristics. The analysis is conducted using a lumped mass system of magnetic particles. Instabilities caused by the magnetic interactions are exploited in combination with particle contact to produce topological changes leading to stiffness changes and corresponding variations in wave propagation patterns. The propagation of plane waves is predicted by applying Bloch theorem to linearized unit cells, and subsequently evaluated through direct numerical simulations of the fully nonlinear governing equations. Changes in mechanical properties are also assessed through homogenization procedures relying on the long-wavelength approximation of the discrete equations of motion and the formulation of equivalent continua.

57. **Gal Shmuel,** galshm@caltech.edu
Co-authors: Adam Thor Thorgeirsson, Kaushik Bhattacharya
Wavelet Analysis of Microscale Strains

Recent experiments and numerical simulations provide numerous observations on the microstructure and deformation of polycrystals. These show how confined bands of deformation percolate in a complex way across various grains. Such information is represented as samples on grids, and, in turn, creates huge data sets. The extensive size of data in this form renders identifying key features difficult, and the cost of digital storage expensive. To represent, analyze, and predict strain fields with localized features, we use wavelets: multiresolution functions, which are localized both in frequency and real domains. By way of example, we focus on pseudo-elastic polycrystals, capable of recovering strains beyond an apparent elastic limit. We will show how wavelets efficiently represent experimental and simulated strains of Ni-Ti, while reducing data size by two orders of magnitude. More importantly, We will show how the compact wavelet representation captures the essential physics within.

58. **Alexey Stepanov,** alex314@umd.edu
Co-authors: Stuart S. Antman
Equilibrium States of Nonlinearly Elastic Annuli and Spherical Shells

Within the linear theory of elasticity, the problems for the equilibria of circular annuli and spherical shells composed of homogeneous, transversely isotropic materials were solved by Gabriel Lamé. The radially symmetric equilibria of an isotropic nonlinearly elastic disk or ball is elementary. If, however, the disk or ball is aeolotropic, even for a homogeneous linearly elastic material, the solution can exhibit a rich range of singular behavior at the origin. We show that BVPs for the equilibria of circular annuli and spherical shells composed of transversely isotropic nonlinearly elastic materials are far from elementary within the framework of geometrically exact theories. We employ a variety of mathematical approaches, discussing the virtues and idiosyncracies of each.

59. **Tuomas Tallinen, tuomas.**`tallinen@jyu.fi`
Hyperelastic modeling of morphogenesis

Soft tissues in developing embryos exhibit complex mechanics with characteristics between those of fluid and solid. Elastic behavior of the tissues at short time scales is followed by inelastic deformation and flow, and interesting open questions are related to the underlying cellular dynamics. For example, tissue fluidity may result either from flow of cells or from nonuniform cell proliferation. We model growing soft tissues by continuum mechanics, starting from a hyperelastic model, with extensions to inelasticity and flow. In particular, we first focus on the development of the gut and its patterning through mechanical instabilities. Even though the endoderm and mesenchyme tissue layers are just few cells thick, a minimal elastic continuum model provides a reasonable description of the mechanical behavior. We also consider the folding of the brain and how its neuronal and axonal growth may be mapped to a large strain visoelastic model producing folding patterns consistent with observations on a wide range of brains.

60. **Andrew Thaler,** thaler@math.utah.edu
Co-authors: Graeme Milton
Bounds on the volume fraction of an inclusion in a body with complex conductivities

In this talk, I will present some recent work concerning the derivation of bounds on the volume fraction occupied by an inclusion in a body (or one of the phases in a two-phase composite). We assume that both the inclusion and the body are isotropic and homogeneous with complex-valued conductivities. The bounds we derive correlate the average electric field, average current field, and volume fractions of the phases; practically, the average electric field and average current field can be measured in terms of boundary measurements of the complex potential and electric current flux. These bounds could have potential applications in nondestructive testing and medicine, such as in the screening of organs prior to transplantation.

61. **Andrejs Treibergs,** treiberg@math.utah.edu
Co-authors: Marc Briane, Graeme W. Milton
On the Realizability of Electric Fields in Conducting Materials

Given an electric field ∇u in $\mathbf{R}^d$, we discuss whether it is realizable in the sense that there is a conductivity matrix function σ such that the current field given by Ohm's law, $\sigma\nabla u$, satisfies conductivity equation

$$\operatorname{div}(\sigma\nabla u) = 0.$$

For application we focus on periodic fields. In the isotropic case the conductivity, given by a scalar $\sigma = fI$, may be found by solving a first order partial differential equation. A smooth, non-vanishing, electric field may be isotropically realized locally. Assuming that the vector field is periodic or satisfies a geometric condition such as having a transverse hyperplane, it may be realized globally. If the electric field is not smooth, it may not not be realized. If the electric field is periodic it may not be realizable by a periodic conductivity. In two dimensions, if a periodic field is realized by a periodic conductivity, then the field does not vanish. This fails in three dimensions. In two dimensions we characterize the periodic gradient fields which are realized by a periodic symmetric positive definite matrix valued function σ. We consider also vector valued potentials and their matrix valued fields DU. Positivity of the determinant is sufficient for a smooth potentials with periodic DU to be anisotropically realizable. The condition is necessary in dimension two. Analogously, laminate electric fields are anisotropically realizable if they have constant sign determinants. [*v.*, "Which electric fields are realizable in conducting materials?," *ESAIM: Mathematical Modeling and Numerical Analysis 2013/89: Special Issue 2014: Multiscale Problems and Techniques*, Archive http://arxiv.org/abs/1301.1613.]

62. **Lev Truskinovsky,** trusk@lms.polytechnique.fr
Passive mechanical behavior of skeletal muscles

In contrast to inert matter, distributed biological systems are characterized by hierarchical network architecture with domineering long-range interactions. Even in the absence of metabolic fuel this leads to a highly cooperative passive mechanical behavior. Collective effects are usually associated with conformational changes which can be interpreted as folding-unfolding transitions. In the presence of long-range interactions systems with internal unfolding exhibit coherent macroscopic hopping between folded and unfolded configurations resisting the destabilizing effect of finite temperatures. A minimal mechanical system exemplifying such behavior is a parallel bundle of bi-stable units linked by two shared backbones and its most well known biological prototype is the collective power-stroke in sarcomeric acto-myosin networks. In this talk we review the mechanics of this mean-field type model at zero and finite temperatures. We show that at zero temperature the ground states always correspond to coherent configurations. The relaxed potential, representing the global minimum of the energy, is convex in a soft device but is only convex-concave in a hard device. Such ensemble non-equivalence and the meta-material response, which persist in the continuum limit, is a result of non-additivity: each element is linked with all other elements and the whole is not a sum of the parts. An important feature of the finite temperature behavior in this system is the appearance of a critical point of Curie-Weiss type separating the correlated hopping at low temperatures from the uncorrelated fluctuations at high temperatures. We show that the actual skeletal muscles appear to be functioning very close to this critical point.

63. **Anna Vainchtein,** aav4@pitt.edu
The discrete charm of nonlinearity: solitary waves in non-integrable lattices

The interplay between discreteness and nonlinearity in many physical systems leads to the formation of solitary waves. For example, such waves were experimentally observed in granular materials, electrical transmission lines and optical fibers. Much of the interest in these nonlinear waves was triggered by the pioneering study by Fermi, Pasta and Ulam (1955). The subsequent work of Zabusky and Kruskal (1965) has revolutionized the nonlinear science by connecting the FPU problem to its quasicontinuum near-sonic limit described by the KdV equation. In integrable systems solitary waves, known as solitons, are now well understood, with one-dimensional Toda lattice being the most prominent example that has an exact solution covering a broad range of behaviors from delocalized low-energy waves in the KdV limit to highly localized high-energy waves. Most discrete systems, however, are non-integrable. In this case understanding the transition from the KdV limit to the strongly discrete waves has mostly relied on numerical and quasicontinuum approximations. In this talk I will review some of these results and describe recent work with Lev Truskinovsky on a non-integrable FPU problem with piecewise quadratic potential. We construct an exact solitary wave solution that captures the entire crossover velocity range between the low-energy limit and strongly localized waves that involve only one particle moving at a time. The solution is expressed in the form of an infinite series. A truncation of the series involving progressively smaller characteristic wavelengths produces a nested set of approximate solutions. Even the simplest solution of this type that accounts only for the longest wave lengths provides a better overall approximation of solitary waves than some conventional quasicontinuum models. Some related open problems will also be discussed.

64. **Valy Vardeny,** val@physics.utah.edu
Co-authors: R. C. Polson
Advances in Organic Random Lasers

Systems which produce laser emission that contain sharp coherent resonant alines, but without an engineered cavity are grouped under the term 'random lasers' (RL). There are many examples of RL systems such as: powders of laser crystals, dye and scatterers in suspension, clusters of nanoparticles, π-conjugated polymer, etc.; here we focus on RL organic gain media. This includes thin films, dye-infiltrated opals, and dye infiltrated human tissues. Usually RL emission spectra contain numerous narrow lines (say > 20) on top of a broader background of amplified spontaneous emission. Since there are so many resonant lines, subsequent analysis of the RL emission spectrum is rather complicated; however it has been relatively easy to explain RL spectra by several different theoretical models that produce a variety of lines, which typically are not correlated one to the other. This type of theoretical view misses an important experimental discovery, namely that the average Fourier transform (FT) of RL spectra over different locations on the organic gain medium film does not average out to a smooth line, but instead contains regular Fourier components that are typical to the material studied and the excitation intensity. We show that this phenomenon stems from the existence of naturally formed microcavities in the gain medium films, which can be effectively mapped using a novel optical technique. We also show that the FT spectrum of RL emission from dye infiltrated human tissues can be used for cancer diagnosis and mapping.

65. **Martin Wegener,** `martin.wegener@kit.edu`
Experiments on Elastic and Thermodynamic Cloaking

We review our recently published as well as unpublished experiments on cloaking in optics, mechanics, and thermodynamics. This includes broadband cloaking of flexural waves in thin elastic plates based on coordinate transformations, three-dimensional core-shell elastostatic cloaks based on pentamode metamaterials, transient thermal cloaks based on coordinate transformations, and three-dimensional cylindrical as well as spherical core-shell invisibility cloaks for diffusive light that work throughout the entire visible spectral range.

66. **Niklas Wellander,** niklas.wellander@hotmail.com, niklas.wellander@foi.se
Co-authors: Gerhard Kristensson
A priori estimates of solutions to the Maxwell equations

We present a collection of a priori estimates of the electromagnetic field scattered by a general bounded domain. The constitutive relations of the scatterer are in general anisotropic. Surface averages are investigated and several results on the decay of these averages are presented. The norm of the exterior Calderón operator for a sphere is investigated and depicted as a function of the frequency.

67. **Anna Zemlyanova,** azem@math.ksu.edu
Curvature-dependent surface tension in modelling of fracture

A new model of fracture mechanics which takes into account interfacial effects due to a curvature-dependent surface tension will be considered. This model is based on a physically valid assumption that the behavior of molecules near a surface of a material is significantly different from those in the bulk and depends on the local curvature of the material surface. The theory will be presented through several examples: a curvilinear non-interface and interface crack, and contact problems for a rigid stamp indentation into an elastic half-plane. It will be shown that the incorporation of surface effects on the crack boundary eliminates the power and oscillating singularities at the crack tips which are predicted by linear elastic fracture mechanics. The mechanical problems will be reduced to the systems of singular integro-differential equations. The regularization and numerical solution of these systems will be addressed and numerical examples will be presented. Potential direction for future research and connections with experimental results will be discussed.