5710 Fall 2013. Home Work 1

August 28, 2013

1. Consider an equation $Ax = b$ where

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ -1 & 1 & 2 & 3 \\ 2 & 0 & -1 & -2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix},$$

Show that A^{-1} does not exists, find the least square solution with the regilarizing term $\|x - a\|^2$, where $a^T = (1, \, 0 \, 0 \, 1)$.

2. Consider a quadratic form $Q = x^T A X$ where

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Represent it as a sum of squares

$$Q = \sum_{i=1}^{4} c_i y_i^2$$

where $y = Lx$ is a linear transformation of the coordinates, and L is a triangular matrix. Is the form positively defined?