m-3150-001 Midterm exam 2

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Your name, ...

Solve any three problems:

1. Find the displacement $u(x,t)$ of a string of a length 5, if its initial displacement $u(x,0)$ is

$$u(x,0) = \sin\left(\frac{\pi x}{5}\right) + \frac{1}{3}\sin\left(\frac{3\pi x}{5}\right),$$

its initial speed is zero. and the spring constant c is $c = 2$.

2. Initially, a rod has zero temperature $u(x, 0) = 0$. Its left end $(x = 0)$ is kept at zero temperature $u(0, t) = 0$ and its right end $(x = 2)$ – at the temperature 100, $u(2, t) = 100$, and the diffusivity constant c is equal to one, $c = 1$. Find the temperature inside the rod at $t = 2$. (Hint: Account for nonzero boundary conditions)

3. The left end of a rod is thermo-insulated, and the right end is kept at zero temperature.

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = 0, \quad u(3,t) = 0, \quad \forall t \in [0, \infty).$$

Find a general solution to the heat problem, assuming that the initial temperature is given, $u(x,0) = f(x)$ and the diffusivity constant c is equal to one, $c = 1$.

4. Separate variables in the the two-dimensional heat equation *(proceed similarly to the two-dimensional wave equation analysis)*. Show the scheme of variables separation, resulting differential differential equations, general soultion, do *not* compute coefficients.

$$\frac{\partial u(x,y,t)}{\partial t} = c^2 \left(\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} \right)$$

if $0 \le x \le a, \quad 0 \le y \le b$, $u(x,y,0) = f(x,y)$,

$u(0,y,t) = u(a,y,t) = 0$,

$u(x,0,t) = u(x,b,t) = 0$