Hint to cloaking problem

1. The problem: A circular inclusion of conductivity σ_{inc} of radius p is placed in an infinite plane of conductivity σ_0 and a homogeneous field $u(r, \theta)$ is applied at infinity,

$$\lim_{r \to \infty} u(r, \theta) = A_0\, x = A_0 r \cos(\theta)$$

where A_0 is a known constant. Create a *cloaking device* placing an annulus (ring) $p < r \leq q$ of conductivity σ_{cl} around the inclusion, so that the field u outside of the cloaking region $r > q$ becomes equal to an undisturbed $u(r, \theta) = A_0 r \cos(\theta)$.

2. The field $u(r, \theta)$ in the outside domain, cloaking domain, and the inclusion satisfies Laplace equation and is equal to:

$$u(r, \theta) = \left(A_0 r + \tfrac{B_0}{r}\right) \cos(\theta) \qquad \text{if } r > q \tag{1}$$

$$u(r, \theta) = \left(A_{cl} r + \tfrac{B_{cl}}{r}\right) \cos(\theta) \qquad \text{if } p < r \leq q \tag{2}$$

$$u(r, \theta) = (A_{inc} r) \cos(\theta) \qquad \text{if } r < p \tag{3}$$

$$\tag{4}$$

respectively, where $B_0, A_{cl}, B_{cl}, A_{inc}$ are some constants.

3. At the boundaries $r = p$ and $r = q$, the following continuity conditions are satisfied:

A. The potential is continuous $u_+ = u_-$, where u_+ and u_- are the values of potential to the left and to the right of the jump line.

B. The normal current is continuous:

$$\sigma_+ \left.\frac{\partial u}{\partial r}\right|_+ = \sigma_- \left.\frac{\partial u}{\partial r}\right|_-$$

These four relations allow for finding the unknown constants $B_0, A_{cl}, B_{cl}, A_{inc}$ if p, q are known.

4. To create a cloak, find the radius q so that $B_0 = 0$ which will make the outside field $r > q$ undisturbed, as if there were no inclusion.