History of Math - M 3010
Introduction

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Fall semester, 2021

1 Mathematics today

1.1 Mathematics and Natural Sciences

The history of mathematics is unlike the history of other natural sciences. Natural sciences develop within paradigms that alternate with time. For example, DNA discovery (James Watson and Francis Crick, 1953) rebuilt the very foundation of biology. The discovery of atoms' structure (Niels Bohr, 1905, and quantum theory in the 1930s) was followed by rethinking chemistry basics. The discovery of plate tectonics (the 1950s) changed the foundation of geophysics. Today's picture of the Universe is much different from what we imagined 70 years ago, and so on. Studying the earlier stages of development of natural sciences is mainly a subject of history departments, not the College of Sciences. For example, al-chemistry and astrology are curious, but these theories have little to do with modern chemistry and astronomy.

Math discoveries are timeless In contrast, mathematics only adds to its treasure of knowledge; it develops and generalizes the previous findings but does not reject them. In this aspect, math is similar to art and especially to technology. We admire ancient Greek sculptures as we admire Greek geometry. Moreover, we teach this geometry in middle school. Likewise, we use the antique wheel and level today and will do so in the future.

Mathematics collected an enormous body of concepts and theories in the four millennia of its development. The challenge of this course is to overlook their origins.

1.2 The size of Math research

Every year, there are approx 120,000 mathematical research papers published each year (MathSciNet and Scimago).

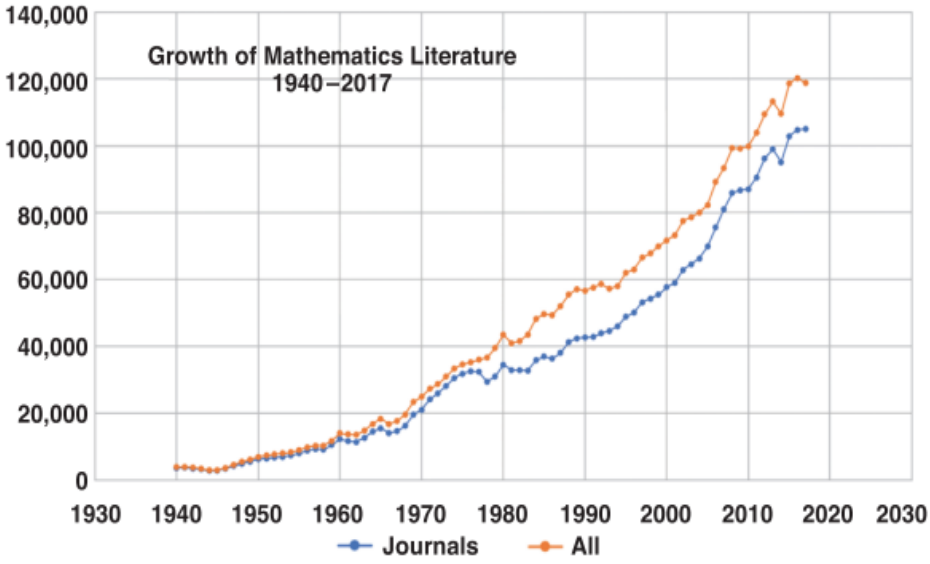

Figure 1: Map of Mathematics

There are tens of thousands of active research mathematicians in the world, see *Looking at the Mathematics Literature* by Edward Dunne

Glossary of areas of mathematics see
en.wikipedia.org/wiki/Glossary_of_areas_of_mathematics

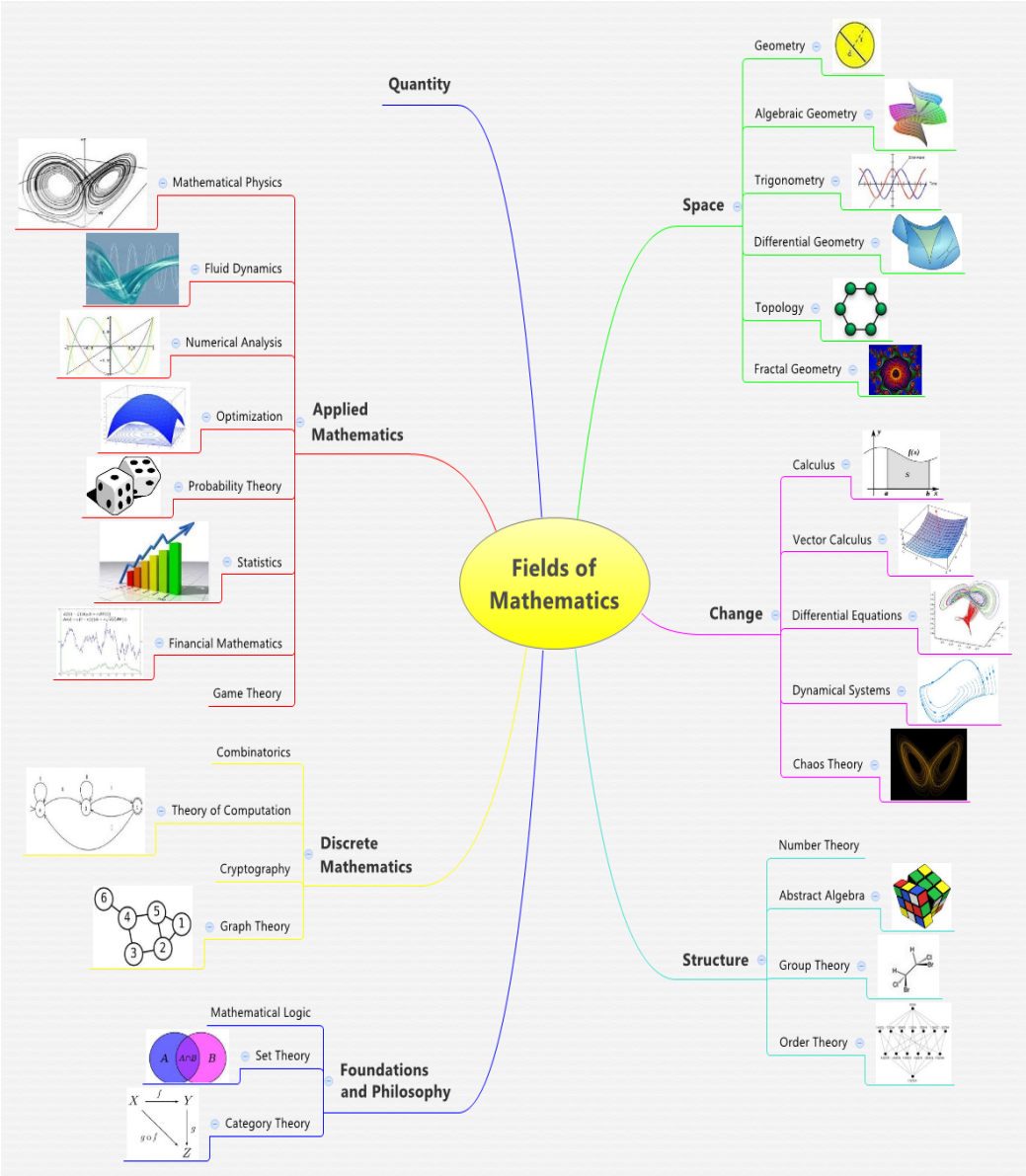

Figure 2: Map of Mathematics

2 How did we get there? Timeline of mathematics

To warm-up for the course, browse through the sites:

– en.wikipedia.org/wiki/Timeline of mathematics

– https://www.storyofmathematics.com

3 Syllabus

3.1 Anchient Mathematics

Prehistory: Math originated in the cradles of civilization: Sumer-Babylon, Egypt, The Indus Valley, China, England, etc. Almost all available documents come from Babylon and Egypt.

Bronze Age: Sumer-Babylon, and Egypt: Base 60, multiplication and division, a geometrical solution of quadratic equations. Approximation of Pi, Pythagorean triples, the iterative algorithms for square roots, binary system, Egyptian fractions.

Greek Math: Pythagoras and this society, Archimedes and beginning of calculus, Euclid - geometry and number theory, Ptolemy - a model of the Solar system.

Chinese, Indian, and Islam math: the story of zero, trigonometry, algorithms, etc

Europe in the 16th century: works of Copernicus, Napier logarithms, Tartaglia, Cardano. Solution of the cubic equations and introduction of complex numbers. Works of Galileo, Kepler.

Students will write essays on the topics of ancient math. The paper should have a form of a story suitable for middle or high school students.

3.2 Modern Mathematics

- formed in the 17th century (during the scientific revolution) [calculus, coordinates] and developed in the 18th century
- hugely expanded in the 19th century (the first industrial revolution)
- greatly expanded again in the 20th century (the second industrial revolution)
- is enormously growing in the last 50 years (computerization)

The 17th-century mathematics: Fermat, Newton, Leibniz, Descartes, Pascal, Huygens: Calculus, functions, coordinates, differential equations, probability

The 18th-century mathematics Bernoulli family, Euler, Lagrange: Calculus of variations, series, algebraic notations, modeling of nature, topology. complex-valued functions

Nineteen century

Analysis, Fourier series, stability
Algebra: Solvability of equations, Group theory
Vectors and Matrices
Partial differential equations.
Curved spaces, non-Euclidean geometry,
Generalization of functions, divergent series,
Set theory, paradoxes

Women in math Maria Agnesi, Sophie Germain, Sofia Kovalevskaya, Emmy' Noether, Marian Mirzakhani, end others.

Faces and dramas behind theorems: Lagrange, Laplace, Sophie Germain Cauchy, Abel, Galois, Gauss, Lobachevsky, Riemann, Weierstrass, Chebyshev, Agrand, Hamilton, Moebius, Maxwell, Cayley, Sylvester, Heaviside, Kelvin.

Glance into contemporary mathematics (20-21 centuries) Foundations: Cantor - Continuum hypothesis, Goedel theorem.

Analysis: Hilbert space, Distributions, Measures, and integration.
Probability and Statistics
Geometry: Fractals, Penrose tiling, graphs. Computer-aided proofs.
Representation theory. Algebraic geometry. Number theory.
Differential equations: Bifurcations, blow up, stochastic.
Aggregation: chaos and homogenization. The math of conflict: game theory.
Math modeling. Optimization.
Statistics and Big Data.
Numerics. Computation and Machine Learning.

What's next?

4 Discussion

How to orient in the enormous contemporaty math literature?

How can we effectively communicate math discoveries?

How to preserve the knowledge, and what should be kept?