

m 5500 Calculus of Variations: Syllabus.

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1. Introduction: History. Minimization Problems. Methods.
2. First variation. Applications to physics, geometry, and approximations.
 - (a) Euler equation. Smooth approximations, geometric optics.
 - (b) Euler-Lagrange equation (multiple integrals). Variational principles: Conductivity, elasticity.
 - (c) Variation of boundary terms. Examples (distance to a surface, cooling boundary conditions, extreme loading)
 - (d) Constrained minimum. Lagrange multipliers (isoperimetric and pointwise constraints). Introduction to Lagrangian mechanics.
 - (e) Variation of the domains. Examples from geometry and physics.
3. Legendre, Jacobi, Weierstrass conditions for one- and multivariable integrals. Ill-posed problems, regularization and relaxation. Examples.
4. Convexity. Duality. Legendre transform. Dual variational principles. Hamiltonian.
5. Minimizing sequences and numerical methods. Numerical example.
6. Existence and uniqueness theorems. Overview.
7. Nonconvex problems. Oscillating minimizers. Convex envelope.
8. Quasiconvexity and the bounds Two examples of relaxation.