HW 2

m 5500 Calculus of variations, Spring 2013

1. Reformulate the minimal surface of revolution problem as a problem for an optimal parametric curve $x(t), y(t)$ ($x(t)$ and $y(t)$ are two minimizers). Solve, show the catenoid and Goldschmidt solutions.

2. The thermal equilibrium is described by the boundary value problem for the temperature $T(x)$

$$\delta T = \gamma(x) T \text{ in } \Omega, \quad \frac{\partial T}{\partial n} + \alpha T^4 = T_0 \text{ on } \partial\Omega,$$

   where γ is the density of heat sources, α is the radiation constant, T_0 is a constant outside temperature.

   Write the variational problem which minimizer describes the equilibrium.

3. Find the Euler-Lagrange equation:

$$I = \min_{u: u|_{\partial\Omega} = u_0} \int_\Omega |\nabla u|^p dx, \quad \geq 1$$

   Consider the case when $p \to \infty$.