# m5500 Spring 2015 HW2

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## Approximations

1. Approximate the functions:

$$a)\quad f_1(x) = \begin{cases} 1 & \text{if } x \in \left[0, \frac{1}{2}\right) \\ 0 & \text{if } x \in \left[\frac{1}{2}, 1\right) \end{cases} \qquad f_2(x) = x^6. \quad x \in [0,1]$$

with penalties

$$p_1 = \epsilon \int_0^1 (u')^2 dx \text{ and } p_2 = \epsilon \int_o^1 (u'')^2 dx$$

Using Maple or Matlab, plot the graphs of the approximates for different $\epsilon$.

*Hint:* Use Fourier series to represent $f_1$ and $f_2$ and to represent the approximate. Compute the coefficients of the approximate.

2. Approximate $f(x) = \sin(x)$, $x \in [0, 2\pi]$ using the penalty functional

$$P = \epsilon \int_0^{2\pi} F_\delta(u') \, dx$$

where

$$F_\delta(x) = \begin{cases} |x| & \text{if } |x| \geq \delta \\ \frac{1}{2\delta}x^2 + \frac{\delta}{2} & \text{if } |x| \leq \delta \end{cases}$$

Plot the approximates.

*Hint:* Consider three cases: $u' > \delta$, $u' < -\delta$, and $|u'| < \delta$.