CalcVar HW 2

Due Monday February 3

1. Solve
$$\min_{u(x)} \int_0^1 \left(k(x)(u')^2 - \omega^2 u^2\right) dx, \quad u(0) = u(1) = 1$$
where
$$k(x) = \begin{cases} k_1, & x < a, \\ k_2, & x > 2. \end{cases}$$

2.
$$\min_{u(x),b} \int_0^b \left(u'^2 + x^2 u\right) dx, \quad u(0) = 0, u(b) = 1$$

3. Derive Euler-Lagrange equation and variational boundary conditions for
$$\int_\Omega \left[(\nabla u - A)^2 + \frac{1}{u}\right] dx$$
where Ω is a bounded domain with smooth boundary, and $A = A(x)$ is a given vector field.

4. Knowing that the Euler-Lagrange equation is the boundary-value problem (cooling of the black body)
$$\nabla^2 u = 0 \text{ in } \Omega, \quad \frac{\partial u}{\partial n} + \alpha u^4 = 0 \text{ on } \partial\Omega,$$
where n is the normal to the boundary $\partial\Omega$, find the Lagrangian.